Numerical simulation of vortical flows in the near field of jets from notched circular nozzles

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A B S T R A C T
The vortex dominated flows in the near field of jets from notched circular nozzles are investigated using direct numerical simulation. The nozzles studied include a normal circular nozzle, a V-shaped notched nozzle, and an A-shaped notched nozzle, all with the same circular cross-section. The vortex structures resulting from these different circular nozzles are visualized by using a numerical dye visualization technique. Results for the V-shaped notched nozzle are compared with available experimental measurements using laser-induced fluorescence techniques. In addition to azimuthal vortex rings created because of the shear-layer between the jet and the ambient fluid, the computations also reveal streamwise vortex pairs both inside and outside the vortex rings that spread outward as the vortex rings move downstream. Comparisons of the three different nozzles show that, unlike in the case of the circular nozzle where the streamwise vortex pairs emerge evenly along the nozzle lip, streamwise vortex pairs for the notched circular nozzles are produced only at peak and trough locations. Analysis of the mixing characteristics of the three types of nozzles shows that the notches in the nozzle exit significantly enhance jet mixing.

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1. Introduction
The mechanism of jet entrainment and mixing is a fundamental flow phenomenon that has direct relevance to many technological applications such as noise suppression, combustion, and heat transfer. Vortex evolution and interaction are the distinctive features of jet flows [1–4]. The three-dimensional vortex development of the jet is particularly sensitive to the initial conditions at the jet outlet. Many experimental and numerical investigations have been performed to examine the important role of the nozzle exit geometry on the vortex evolution. Hussain and Husain [5–7] studied the coherent structures of elliptic jets. Jets from nozzles of other non-circular cross-sections have also been studied [8–11]. Non-circular nozzles are studied in light of flow control purposes. A good review article is given by Gutmark and Grinstein [8]. Compared with active means of jet control, passive means of jet control such as altering the jet exit geometry offers the advantages of simplicity and reliability of the nozzle. An alternative means of jet flow control is to maintain the circular cross-sectional shape of the nozzle but use non-planar nozzle lips (the so-called indeterminate-origin jets) as studied experimentally by Wlezien et al. [12], Longmire et al. [13,14]. They found that the jet flows from the nozzle with stepped and saw-tooth lips depend largely on the vortex rings and their interactions with the streamwise vortices. The formation and the interaction of the streamwise vortices tended to enhance the mixing of the jet with the ambient fluid. Webster et al. [15] and Lim [16] also studied inclined nozzle lips. Recently New et al. [17] provided additional experimental observations of jets from a V-shaped notched nozzle in a water jet facility at low Reynolds numbers. The flow evolution from the V-shaped notched nozzle was studied using a laser-induced fluorescence technique. Video images revealed salient features of the flow at various stages of its development. New et al. [17] proposed a hypothetical vortex line model to illustrate the conjectured flow mechanism based on visual inspection of the images. The lack of quantitative measurements of velocity and vorticity distributions in the jet flow leaves many issues unexplained. While it is unmistakable that the peaks and troughs trigger the formation of the streamwise vortices, it is unclear how and to what extent the peaks and troughs differ in their respective contributions to the formation of such streamwise vortices.

The present paper is an attempt to provide a numerical solution for the jet flow from notched circular nozzles. The Navier–Stokes equations are solved on very fine grids to capture the vortical structures of the flow in the near field of the jet. A numerical dye visualization technique is developed to trace the flow motion in analogy to the experiment of New et al. [17] so as to provide a means to visualize the computed flow field and compare with the experiment. To assess the effects of the different troughs and
peaks a normal circular nozzle (C-nozzle), a V-shaped notched nozzle (V-nozzle), and an A-shaped notched nozzle (A-nozzle) with the same circular cross-section are compared to explore the generation and evolution of the vortices.

2. Numerical method

2.1. Basic numerical algorithm

The Navier–Stokes equations are written as

\[
\frac{\partial}{\partial t} \int \int_V \rho \mathbf{v} \cdot d\mathbf{v} + \int \int_S \mathbf{F} \cdot d\mathbf{n} = 0
\]

where \( V \) is an arbitrary control volume with closed boundary surface \( S \), and \( \mathbf{n} \) is the unit normal vector. The variable vector \( \mathbf{W} \) is defined as \((\rho, \rho u, \rho v, \rho w, \rho e)\), in which \( \rho, u, v, w, \) and \( e \) are the density, the three velocity components, and the total energy, respectively. The flux tensor \( \mathbf{F} \) consists of a convective (inviscid) part and a diffusive (viscous and thermal) part. The coefficient of molecular viscosity is obtained by Sutherland’s formula.

Eq. (1) is solved by using a cell-centered finite-volume method [18]. The solution domain is divided into small cells. Eq. (1) holds for each cell

\[
\frac{\partial}{\partial t} (W \Delta V) = R(W) = -Q + D
\]

where \( R(W) \) is the residual, \( \Delta V \) is the cell volume, \( Q \) is the net flux leaving the cell

\[
Q = \sum_{k=1}^{6} [F_{xk} (\Delta S)_k + F_{yk} (\Delta S)_k + F_{zk} (\Delta S)_k]_k
\]

where \( F_x, F_y, \) and \( F_z \) are the \((x, y, z)\) component of the flux vectors, respectively, in the Navier–Stokes equations, which include both the viscous and inviscid flux contributions; \( k \) indicates the 6 cell faces of a center cell. The flux vectors at each cell face are calculated by using the average of their cell-center values on either side of the cell face. This amounts to a central-difference scheme. In order to remove potential odd–even coupling mode present in a typical central-difference scheme, a fourth-order artificial dissipation term \( D \) is added

\[
D = (d_{i+1/2,j,k} - d_{i-1/2,j,k}) + (d_{j+1/2,i,k} - d_{j-1/2,i,k}) + (d_{k+1/2,i,j} - d_{k-1/2,i,j})
\]

where \((i, j, k)\) denotes the current cell center, the 1/2 indices indicate the cell faces, the dissipation flux vectors \( d \) are then formulated by a third-order difference in each grid direction, for example,

\[
d_{i+1/2,j,k} = e_{x_{i+1/2,j,k}} R_{i+1/2,j,k} (W_{i+1/2,j,k} - 3W_{i+1,j,k} + 3W_{i,j,k} - W_{i-1,j,k})
\]

where \( e_{x_{i+1/2,j,k}} \) is an adjustable coefficient \( R_{i+1/2,j,k} \) is a proper scaling factor proportional to the cell face area and the wave speed of the flow. The artificial dissipation flux term is of third-order. The basic spatial discretization remains second-order accurate. If we discretize the time derivative term by an implicit backward-difference scheme of second-order accuracy, we obtain at time level \( n + 1 \)

\[
\frac{\Delta V}{\Delta t} [3W_{n+1} - 4W_{n} + W_{n-1}] = R(W_{n+1})
\]

In order to obtain a solution for \( W_{n+1} \), an iterative approach has to be applied. We reformulate the problem at each time step as the following steady-state problem in a pseudo-time \( \tau \)

\[
\frac{\partial}{\partial \tau} W_{n+1} = \frac{1}{\Delta V} R(W_{n+1}) - \frac{1}{2\Delta t} [3W_{n+1} - 4W_{n} + W_{n-1}]
\]

The steady-state solution in pseudo-time of the above system is then equivalent to the solution of the time-accurate problem for one physical time step specified by Eq. (3). Therefore, we can apply various acceleration techniques for steady problems to solve Eq. (4). For this purpose, a multi-stage Runge–Kutta scheme is used to integrate Eq. (4). Multi-stage Runge–Kutta schemes are used for their extended stability range, i.e., larger stable CFL numbers together with local pseudo-time stepping and residual smoothing. In addition, the multi-grid method is used to accelerate the convergence effectively. To reduce the calculation time the code is also parallelized by using domain decomposition and MPI to take advantage of parallel computers or clusters of PCs. Details of these techniques are found in Ref. [19].

2.2. Numerical dye visualization method

In analogy to the experimental dye visualization technique, a numerical “dye” concentration \( \theta \) is obtained by solving the following scalar transport equation

\[
\frac{\partial}{\partial t} \int \int_{V} \theta dV + \int \int_{S} (\theta \mathbf{v} + \kappa \nabla \theta) \cdot d\mathbf{n} = 0
\]

where \( \kappa \) is the coefficient of diffusion of the dye and \( \mathbf{v} \) is the flow velocity vector \((u, v, w)\) obtained independently from Eq. (1). For convenience, Eq. (5) is discretized using the same numerical methodology as used above for Eq. (1). The resulting discrete equation, however, can be efficiently solved by an approximation-factorization method (AF method) after the velocity field \( \bar{\mathbf{v}}_{n+1} \) at time level \( n+1 \) is obtained from solving Eq. (3). Thus, we solve

\[
(I + \delta_i)(I + \delta_j)(I + \delta_k) \Delta \theta^d = \bar{R} (\rho^d - \bar{\rho}^{d+1})
\]

where \( \Delta \theta^d = \rho^{d+1} - \bar{\rho}^d \), and \( \delta_i, \delta_j, \) and \( \delta_k \) are cell-face flux difference operators in the \( i, j, \) and \( k \) grid-line directions, respectively. Here the residual \( \bar{R} \) is

\[
\bar{R} = \frac{2\Delta \mathbf{v}}{3\Delta V} R (\rho^d, \bar{\rho}^{d+1}) + \frac{1}{3} (\rho^d - \bar{\rho}^d - \theta)^2
\]

Eq. (6) can be efficiently solved in each direction by using a tridiagonal solver. The convergence of the above implicit AF method is very fast.
2.2.1. Nozzle models, computational grids, and boundary conditions

Three different configurations of nozzles as shown in Fig. 1 are studied. All these nozzles have the same circular cross-sections with an inner diameter of $D$. The reference circular nozzle has a regular planar exit and is here called the C-nozzle. The notched nozzles are constructed by cutting the exit of the regular circular nozzle with two symmetrically-placed cutting planes in the shape of the letter V for the V-shaped nozzle and that of the inverted V for the A-shaped nozzle. The aspect ratio of the notches is defined as $R_n = 2d/D$, where $d$ is the depth of the notch. Both nozzles have the same notch aspect ratio of $R_n = 1.2$. The half-depth cross-plane of the notches is defined as the exit plane for the notched nozzles.

As in the actual experimental setup by New et al. [17], the nozzles are mounted on a horizontal plate (see Fig. 1). All three nozzles have the same height of $2D$ measured from the horizontal plate to the exit plane along the axial direction $x$. The origin of $x$ is fixed at the exit plane. The nozzle inlet extends upstream the horizontal plate by $3D$. Boersma et al. [20] studied the influence of the shape of the velocity profile at the jet exit on the self-similarity scaling for the far-field turbulent velocity and shear stress profiles. The present studies are focused on the effect of the nozzle exit geometry on the jet flow. The nozzle is modeled by a zero-thickness solid wall. Uniform flow condition is specified at the inlet to the nozzle. No-slip wall boundary conditions are used on both the inner and outer surfaces of the nozzle so that the flow will develop inside the nozzle into a natural viscous velocity profile at the nozzle exit before it mixes with the ambient flow external to the nozzle downstream of the horizontal separation plate. The Reynolds number is defined as $Re = UD/\nu$, where $U$ is the mass-averaged velocity at the exit of the nozzle and $\nu$ is the kinematic viscosity of the fluid. The computational domain excludes the space external to the nozzle and upstream the separation plate, but includes the complete internal nozzle region and the space external of the nozzle and downstream of the separation plate. The external region behind the separation plate provides the needed 'buffer' region that was found important for the near-field behavior of jet flows by Babu and Mahesh [21].

The downstream and side far-field boundaries of the computational domain are located at $16D$ downstream the nozzle exit and $12D$ from the nozzle axis, respectively. Very fine grids are used in the computation to ensure that the vortex structures are adequately resolved. Fig. 2 shows the topology of the multi-blocks of the body-conforming grid for the V-nozzle. Computations of the C-nozzle are performed with a grid that covers the full $360^\circ$ of the azimuth. A total of 18 blocks are used to cover the complete computational domain with a total of $193 \times 115 \times 193$ (~4.28 million) grid points in the axial, radial, and azimuthal directions, respectively. The grid is clustered near the nozzle boundary to capture the high shear in that region. The radial grid resolution for all the three nozzles is identical. The grid distributions in the axial direction are made to conform to the different nozzle lip for each nozzle.
of the nozzles. In the case of the notched nozzles, the presence of the peaks and troughs appear to impose asymmetry to the flow, as evidenced in the experimental observations by New et al. [17] and also confirmed by our initial computations of the V-shaped nozzle using a grid covering the full domain. Because of this symmetry, we opted to use only one quarter of the domain along the azimuthal direction from trough to peak for the notched nozzles in order to save computational resources. A grid convergence study was performed for the V-nozzle. The above mentioned grid is determined by assuring that the resolved flow features remain essentially unchanged when the grid is further refined. No mushroom type of structures appeared on the coarse grid computations. It was only on the fine grids when these structures appeared and became grid independent.

Outside the nozzle, no-slip boundary condition is applied at the nozzle external wall and the horizontal plate where the nozzle is mounted. This plate functions as the upstream boundary for the ambient fluid flow. At the downstream and lateral far-field boundaries, the pressure is specified to be that of the undisturbed ambient fluid, while other flow variables are extrapolated from the interior points. Symmetry boundary conditions are used at the grid boundaries in the azimuthal direction both inside and outside the nozzle. At the inlet boundary of the nozzle, the dye concentration is set to 1.0. It was extrapolated at all other boundaries of the computational domain.

3. Computed flow structures and mixing characteristics

Unsteady vortex flows of the jets are calculated. Initially, uniform flow velocities and dye concentration that are the same as those at the inlet of the nozzle are set inside the nozzle and in the extruded cylindrical volume from the nozzle exit to the downstream boundary of the computational domain. The velocities and dye concentration in all other areas are set to 0. The pressure $p$ and density $\rho$ are, however, set to the undisturbed ambient values throughout the computational domain. After a relatively large time level ($t > 32T$), the vortex flow of the jets assumes a stable periodic motion. Here $T = D/U$ is the nominal residence time of the jet. To ensure independence from the initial flow conditions, all results shown below are those obtained at $t = 64T$.

3.1. Flow from the C-nozzle

Computations of the C-nozzle are performed with a grid that covers the full $360^\circ$ of the azimuth. Since there is no geometric non-uniformity for the reference C-nozzle, the jet flow is expected to be axisymmetric when there are no intrinsic three-dimensional instabilities at relatively low Reynolds numbers. Indeed, computations of the C-nozzle yields perfect steady axisymmetric flow fields with no non-uniformities in the azimuthal direction of the jet for Reynolds numbers below 2000. At $Re = 2800$ based on the mass-averaged velocity at the nozzle exit, however, unsteadiness appears in the computations and the computed flow soon reaches a stable periodic solution with clear non-uniform flow structures in the azimuthal direction. Fig. 3 shows the instantaneous dye concentration distribution in the $x$-$y$ and $x$-$z$ meridional planes parallel to the jet axis at $t = 64T$. In this figure, $x/D$ ranges from 2 to 10 and the base dimension of the dye distribution is the diameter of the nozzle $D$. The figure shows obvious rib-like streamwise features. These rib-like structures are the same as those observed by New et al. [17]. Instability of the vortex sheet of the jet causes roll-up of the vortex sheet and the formation of concentrated vortex rings at the roll-up locations. Starting with the second vortex ring, the dye concentration spread out from the structures at the roll-up locations. The spread of the dye indicates the initiation and generation of the streamwise vortex pairs observed in experiments by Liepmann and Gharib [3]. The rib-like structures of the dye disappear after four generations downstream. The images in this figure are instantaneous snapshots of the periodically changing dye concentration contour maps. They provide a representative view of the flow structure.

Fig. 4 shows the instantaneous dye concentration overlapped with the instantaneous streamlines at the same time in the half-domain of the $x$-$y$ and $x$-$z$ meridional planes. The first and second generations of vortex rings are located at $x/D = 3.73$ and $x/D = 5.06$, respectively, where the streamlines circle around the
locations of the dye concentration roll-up. The vortex rings induce the ambient fluid immediately upstream of them to move inwards towards the jet axis and the jet fluid immediately downstream to move outwards from the axis, as is indicated by the directions of the streamlines around the vortex rings. The dark regions of the dye concentration between the vortex roll-up locations and the jet core indicate fluid entrainment from the ambient into the jet by the vortex rings. Globally, the ambient fluid is drawn into the jet core as is shown by the inward direction of the flow in the far outer field.

Fig. 4 shows the instantaneous streamlines in the cross-planes parallel to jet axis for C-nozzle.

Fig. 5 shows the instantaneous dye concentration in cross-planes perpendicular to the jet axis located at $x/D = 2.8$, 3.8, to 7.8, respectively. The dimensions of the images are from $-1.5D$ to $1.5D$ in the horizontal ($y$) direction and from $-1.125D$ to $1.125D$ in the vertical ($z$) direction. At $x/D = 2.8$ and 3.8 the dye distributions remain circular. At $x/D = 4.8$ there are six small mushroom-like structures sprouting out from the edge of the circle. They grow larger downstream from $x/D = 5.8$ to 6.8. Liepmann and Gharib [3] observed similar mushroom-like structures in their jet experiments. The number of such structures reported by them
for the jet at a Reynolds number of ~3000 is between 5 and 6, although they are not as symmetrically distributed along the circumference of the jet as they are in our computational results. This difference of the location of the mushroom-like structures between experiment and computation is most likely due to the fact that small random perturbations of the flow conditions or imperfection of nozzle geometry are unavoidable in the experiment while our computations are performed without any intentionally added perturbations in either the initial flow conditions or nozzle geometry. Development of the instability modes reflects the natural evolution of the unstable flow field under the implicit perturbations by random computer round-off errors in the computations. Notice we are able to use very fine grids particularly in the azimuthal direction since our focus is on the initial development of instability in the near field. This enables us to resolve the detailed initial phase of instability. At the far downstream location $x/D = 7.8$, our computations start to exhibit asymmetric development of the large-scale structures as the instability grows. Further growth of such instability will eventually lead to transition to full turbulence and therefore rather random-like patterns as shown by the previous authors. However, computation of the flow in the fully turbulent regime would need more grids to cover the downstream domain and is outside the focus of the present study.

Fig. 6 shows the instantaneous streamlines overlapped with the dye concentration maps in the quarter domain of the cross-planes perpendicular to jet axis at the same $x/D$ locations as in Fig. 5. The horizontal ($y$) and vertical ($z$) directions are both from $-1.5D$ to $1.5D$. Here the vorticity is defined as $\omega = |\nabla \times \vec{v}|$. The maximum and minimum levels of vorticity are 2.5 and 0.3 corresponding to the colors from blue to white with 100 intervals. Fig. 8 shows the corresponding three-dimensional iso-surfaces of the vorticity distribution between $x/D = 2.5$ and $x/D = 8.5$. There are 16 surfaces each with 75% translucency and at a vorticity level ranging from 1.0 to 2.5. The left figure is viewed in the $z$ direction and the right one is viewed in the $y$ direction. For $x/D < 2.5$ the iso-surfaces essentially form a continuous cylindrical surface, showing the initial shear-layer coming off the nozzle lip. Starting from at $x/D = 2.5$, instability of the vortex sheet leads to the roll-up of the vortex sheet and the formation of the first concentrated vortex ring at about $x/D = 3.73$. Fig. 7a shows a cross-section of the symmetric vortex sheet at $x/D = 2.80$ before the formation of the first vortex ring. At $x/D = 3.80$, the first vortex ring is seen in Fig. 7b as being perfectly circular. Fig. 7c shows a cross-section of the second vortex ring at $x/D = 4.80$ slightly ahead of its center at about $x/D = 5.06$. Although it still appears circular, a close examination of the contour densities reveals azimuthal modes of slight perturbations, signifying the initiation of the second instability. These azimuthal disturbances on the second vortex ring may also be seen in the three-dimensional contour plots of Fig. 8. Further downstream at $x/D = 5.8$, slightly ahead of the third vortex ring, the azimuthal disturbances have developed into clear three-dimensional streamwise vortex structures both inside and outside the vortex ring, as shown in both Figs. 7d and 8. These streamwise vortex structures seem to ride on the vortex ring. The outer streamwise vortices grow in size and spread outward as we move downstream at $x/D = 6.8$ shown in Fig. 7e. These streamwise vortices are responsible for the rib-like structures seen in Figs. 3 and 8. Notice, however, that the spread of the streamwise vortex pairs outside the vortex rings is not seen in Fig. 8 because the vorticity levels of the contours are cut-off at 1.0 in that figure. These streamwise vortices interact with each other and the azimuthal vortex rings, producing fine and complex three-dimensional flow structures and greatly enhancing entrainment and mixing of the ambient fluid with the jet flow. At $x/D = 7.8$ the fourth vortex ring has almost given way to many small streamwise pieces as are shown in Fig. 7f. Further downstream, streamwise vortices dominate.

Fig. 6. Instantaneous streamlines in the cross-sections perpendicular to jet axis for C-nozzle.
The above computed near field flow development of the circular jet is in full agreement with the experimental observations of circular jets by Liepmann and Gharib [3]. The development of the three-dimensional streamwise vortex structures is similar to that of the Bernal–Roshko structures in plane mixing layers [22] and is due to the growth of the secondary azimuthal instability modes of the vortex rings. Despite the use of a full annular computational domain for the C-nozzle, our computations do not show the appearance of the helical instability modes as found in the experiments by Yoda et al. [4]. This may be due to the fact that measurements by Yoda et al. [4] were on the far field of the circular jet ($x/D$ from 30 to 80) and at a higher Reynolds number (5000), while the present studies are focused on the near field ($x/D < 10$) and at lower Reynolds numbers. For the same reason, we also do not expect to observe any helical instability modes in the computations of the notched nozzles. Hence, in the following studies of the V- and A-shaped notched nozzles, only one-quarter of the full annular domain is used based on the fact that both our initial computations on the full domain and the experimental data of the V-nozzle by New et al. [17] showed a prevailing symmetry in the azimuthal direction imposed by the peaks and troughs of the notched nozzle. New et al. [17] did not observe any helical modes in their experiments of the V-shaped nozzle in the near field.

3.2. Flow from the V-nozzle

Fig. 9 shows the instantaneous dye distribution in the $x$–$y$ (peak-to-peak) and the $x$–$z$ (trough-to-trough) meridional planes of the V-nozzle at $t = 64T$. Both the time level and the dimensions of this figure are the same as those of Fig. 3. Qualitatively similar rib-like streamwise structures to those shown in Fig. 3 are observed. Like in the case of the C-nozzle, the spread of the dye indicates the initiation and generation of streamwise vortex pairs, but the spread of the dye in the V-nozzle is significantly greater than that in the C-nozzle. Clearly, the V-nozzle enhances mixing of the jet flow.

Fig. 10 shows the instantaneous dye concentration and streamlines in the half domain of the $x$–$y$ (peak-to-peak) and $x$–$z$ (trough-to-trough) meridional planes. The first and second generations of vortex rings are located at $x/D \approx 3.55–3.58$ and $x/D \approx 4.73–4.77$, respectively. The streamlines make complete circles around the first vortex ring but only half circles around the second vortex ring because of the strong outward spread of the jet flow starting from the second vortex ring. The first vortex ring induces an outward motion of the jet fluid, while the second vortex ring induces strong...
entrainment of the ambient fluid towards the jet flow. This counter motion enhances mixing and might also be responsible for the instability of the vortex rings because of the mutual interaction and the strong shear between them.

Fig. 11 shows the instantaneous dye concentration in the same cross-planes as in Fig. 5. The jet starts to spread out faster at the trough locations beginning at $x/D = 2.8$, resulting in an elliptic cross-section of the jet core. Rapid outward spread of the jet can be seen both at peak and trough locations at $x/D = 3.8$ and 4.8. This distinct distortion of the originally circular vortex sheet by the notches of the nozzle exit promotes the distortion and subsequent instability of the vortex rings, leading to the creation of strong streamwise vortex pairs at both the peak and trough locations.

The streamwise vortex pairs induce outward and inward motions of the fluid in different azimuthal regions of the jet. Distinctive grid-like dark regions of the dye concentration inside the jet at $x/D = 4.8$ and 5.8 and subsequent breakup and smearing of them at $x/D = 6.8$ and 7.8 suggest strong entrainment of the ambient fluid into the jet due to the induced velocities by the azimuthal vortex rings and the streamwise vortices. The outer edge of jet core changes from a circle to a square shape. The entrainment and mixing patterns of the ambient fluid with the jet core are very different from those of the C-nozzle shown in Fig. 5. This difference is clearly because of the fact that the troughs and peaks of the nozzle exit fix the primary mode shape of the azimuthal instability of the vortex rings and thus the locations where streamwise vortex pairs appear.

Fig. 9. Instantaneous dye concentration in the cross-planes parallel to jet axis for V-nozzle.

Fig. 10. Instantaneous streamlines in the cross-planes parallel to jet axis for V-nozzle.
Fig. 11. Instantaneous dye distribution in the cross-sections perpendicular to jet axis for V-nozzle; horizontal, peak-to-peak; vertical, trough-to-trough.

(a) $x/D=2.8$
(b) $x/D=3.8$
(c) $x/D=4.8$
(d) $x/D=5.8$
(e) $x/D=6.8$
(f) $x/D=7.8$

Fig. 12. Experiment results of instantaneous laser cross-sections at various downstream locations for V-nozzle (from New et al. [17]).

(a) $x/D=2.0$
(b) $x/D=3.0$
(c) $x/D=4.0$
(d) $x/D=5.0$
Fig. 12 shows the instantaneous dye concentration maps at various downstream locations measured by New et al. [17] using laser-induced fluorescence technique for the same V-nozzle at $Re = 2000$. The experimental and computed patterns shown in Figs. 11 and 12, respectively, are very similar.

Fig. 13 shows the instantaneous streamlines in the cross-planes perpendicular to the jet axis for a quarter of the domain at $x/D = 3.8$, $4.4$, $5.0$ and $5.8$. The white circular line is the projected jet nozzle boundary. The streamlines at $x/D = 3.8$, which is located between the first and second vortex ring (see Fig. 9), are almost along the radial directions. The dye concentration map at this location shows almost a circular jet. There is a circular boundary which separates the outward spreading motion of the jet fluid induced by the first vortex ring and the inward motion of the ambient fluid induced by the second vortex ring. Slightly downstream at $x/D = 4.5$ but still in between the two vortex rings, all the streamlines go inward but are slightly curved. Mushroom-like structures of the dye concentration start to appear. The curved streamlines and the generation of mushroom-like structures indicate the beginning of the instability of the vortex rings and the subsequent generation of streamwise vortices, promoted possibly by the instability of the above circular boundary that separates the opposing fluid motions by the two neighboring vortex rings. The streamwise vortex pairs can be clearly seen at $x/D = 5.0$, slightly downstream of the second vortex ring. The streamlines circle around at the corners of the mushroom-like structures, suggesting significantly enhanced mixing of the jet flow with the ambient fluid. At $x/D = 5.8$ near the third vortex ring, the jet core appears almost as a square. In addition to the streamwise vortex pairs outside the jet core, there are also two counter-rotating streamwise vortex pairs at the corners inside the square jet core. The mechanism of the generation of the streamwise vortices and their effect on the jet for the V-nozzle is the same as that for the C-nozzle. However, the streamwise vortex pairs are found initially only at peaks and troughs for the V-nozzle. The peaks and troughs of the nozzle exit promote and determine the dominating mode shape of the azimuthal instability of the vortex rings.

Fig. 14 shows the instantaneous non-dimensional vorticity contours in the same cross-planes as those in Fig. 11. The dimensions of the images and the contour levels of vorticity are the same as those of Fig. 7. At $x/D = 2.8$ and 3.8 the contours become deformed from the original circular shape and stretch both in trough and peak directions. At downstream cross-sections at $x/D = 4.8$ and 5.8 there are small pairs of ring-like contours (streamwise vortex pairs) riding upon the large vortex ring both inside and outside of the vortex ring. These streamwise vortex pairs are generated only at the trough and peak locations. The large vortex ring breaks into four segments at $x/D = 6.8$ and almost disappears at $x/D = 7.8$. The streamwise vortex pairs become dominant as we move downstream along the jet axis.

Fig. 15 shows the three-dimensional iso-surfaces of vorticity for the flow from the V-nozzle between $x/D = 2.0$ to $x/D = 8.0$. For $x/D < 2.0$ the iso-surfaces of vorticity form circular cylinders, which indicate the initial shear-layer of the jet. The first vortex ring is located at $x/D = 3.52$ and the average distance between the vortex rings is $\Delta x/D \approx 1.23$, a little shorter than that for the C-nozzle, which is $\Delta x/D \approx 1.33$. The first, second and third vortex rings bend at trough and peak locations. The fourth vortex ring breaks into...
four segments downstream and finally evolve into streamwise vortices at $x/D > 8.0$. Streamwise vortex pairs that are generated at the trough and peak locations can be clearly observed. They spread rapidly outward as we move downstream as seen in Fig. 9. However, the streamwise vortices outside the vortex rings are not shown in Fig. 15 because the contour levels in Fig. 15 are cut-off at 1.0 whereas the vorticity levels of the flow are reduced to values lower than 1.0 because of the strong diffusion of the vortices as they spread outward.

New et al. [17] proposed a flow model for the V-nozzle, in which they conjectured a vortex line initially at the nozzle lip and bent down at the trough locations, evolving under convection and self-induction into a corrugated vortex loop with four ridges directly above the peak and trough locations of the nozzle lip. They explained that the bends of the vortex loop at the four ridges gave rise to the streamwise components of the vortices (Figs. 16 and 17 in New et al. [17]). We were not successful in revealing the evolution of such vortex loops from our computational data no matter how we adjusted the vorticity contours in our plots. Instead, our results presented in Fig. 15 show that the flow out of the V-nozzle consists of a uniform cylindrical vortex sheet from the nozzle exit similar to that shown in Fig. 8 for the C-nozzle. The initial vortex sheet rolls up first to form a vortex ring that is only slightly deformed by the presence of the peaks and troughs of the nozzle lip. Further downstream, the vortex rings break into streamwise vortices due to instabilities much in the same way as we discussed in the case of the C-nozzle, except that in the notched nozzle case, the peaks and troughs provide the initial disturbances at fixed locations and therefore appear to pre-determine the mode of the instability of the vortex rings. The results for the A-shaped nozzle to be discussed below show similar flow mechanisms. Notice that the distinctive streamwise vortices are also generated in the C-nozzle case where initial vortex lines at the nozzle exit are perfectly planar. We therefore believe that the flow model by New et al. [17] based on the assumption of an initially bent vortex line in the presence of a continuous vortex sheet is overly simplified.

### 3.3. Flow from the A-nozzle

The lip of the V-nozzle is a sharp corner at its trough location and a rounded edge at the peak location. The A-nozzle reverses the shapes of the troughs and the peaks of the nozzle lip. The troughs are now smooth and the peaks become sharp. Fig. 16 shows the computed instantaneous dye distribution in the $x$-$y$ (trough-to-trough) and the $x$-$z$ (peak-to-peak) planes of the A-nozzle. The dimensions of this figure are the same as those of Fig. 9.
The main rib-like structures of the dye concentration are similar to those of the V-nozzle when the same peak-to-peak and trough-to-trough views of the two nozzles are compared, respectively. Initially, the jet core spreads outwards much more in the trough-to-trough plane than in the peak-to-peak plane. The trend, however, reverses in the downstream region.

Fig. 17 shows the instantaneous dye distribution in the same cross-sections as in Fig. 11. While the initial distortion of the jet of the V-nozzle happens at the trough locations as is shown in Fig. 11, the dye concentration maps at $x/D = 2.8$ and $3.8$ in Fig. 17 indicate that the initial distortion of the circular jet core of the A-nozzle starts at the peak locations. However, we notice that those locations of the two nozzles are the locations where the sharp notches are. Therefore, we may deduce that it is the sharp notches of the nozzle lip that promote the initial distortion and thus subsequent instability of the vortex rings of the jets. The subsequent development of the mixing pattern of the jet appears similar for the two notched nozzles. Again, dark regions of the dye concentration give rise to various patterns of the jet core, suggesting enhanced entrainment and mixing of the jet flow with the ambient...
fluid. The details of the patterns are, however, different for the two notched nozzles.

Fig. 18 shows the instantaneous vorticity contours in the same cross-planes as those of Fig. 14. The dimensions of images and the contour levels are kept the same for the two figures. Consistent with the dye concentration maps of Fig. 17, the vortex sheet at \( x/D = 2.8 \) and the first vortex ring at \( x/D = 3.8 \) show significant distortion at the sharp peak locations. Although the vorticity appears to spread fast across the troughs, streamwise vortices develop first at the peak locations as can be seen from the vorticity contours at \( x/D = 4.8 \). Similar to the V-nozzle, small pairs of streamwise vortex pairs develop and ride upon the large vortex ring inside and outside the vortex ring and at both the trough and peak locations as we go further downstream. The vortex ring again breaks up into four segments at \( x/D = 6.8 \) and almost disappears at \( x/D = 7.8 \). Strong streamwise vortex pairs appear near the two peak locations.

Fig. 19 shows the three-dimensional iso-surfaces of vorticity for the A-nozzle. The exchange of troughs with peaks (from V-nozzle to A-nozzle) does not seem to significantly change the main structures of the vortex rings and the streamwise vortex pairs. The earlier discussions for the V-nozzle apply.

3.4. Mixing characteristics of the three nozzles

Fig. 20 shows the flow velocity along the centerline of the jet. The velocity is normalized by the centerline velocity of the jet at the exit of the nozzle \( U_0 \). The presence of the vortex rings induces local fluctuations of the jet velocity in the near field \( 3.0 < x/D < 9.0 \). The centerline jet velocity is reduced by about 30% in the far downstream at \( x/D = 15.0 \) for the V- and A-nozzles, whereas that for the C-nozzle is only reduced by 10%. The faster reduction of the jet core velocity indicates greater mixing of the jet flow with the ambient fluid for the notched nozzles compared to the regular circular nozzle.

The breakup of vortex rings and the creation of streamwise vortices enhance mixing of the jet with the ambient fluid. Vorticity contours shown in Figs. 14 and 18 for the V- and A-nozzles exhibit axis-switching tendencies, whereas those shown in Fig. 7 for the C-nozzle appear to be always axisymmetric. The axis-switching instability for the notched nozzles may also help enhance entrainment in the near field of the nozzle exit.

In order to further quantify the mixing characteristics of the jet, we define the total amount of dye, \( Dv = \int \int \int \partial dV \), where \( V \) is the cylindrical volume of radius \( r \) and height of 6D along the jet axis between \( 3.0D < x < 9.0D \). \( Dv \) is normalized by \( Dvt \), the total dye in the same cylinder but of radius 10D. For a given radius, a smaller value of \( Dv/Dvt \) indicates that a larger portion of the dye has been spread outside the given cylinder, which means better mixing of the jet. Fig. 21 shows the variation of \( Dv/Dvt \) with radius of the cyl-

![Fig. 18. Instantaneous vorticity contours in the cross-sections perpendicular to jet axis for A-nozzle; horizontal, trough-to-trough; vertical, peak-to-peak.](image-url)
The vortex flow in the near field of jets from a regular circular nozzle and two notched nozzles are computed by solving the unsteady Navier–Stokes equations. A numerical dye concentration is used to visualize the flow field and also provide comparison with previous experimental observations. Azimuthal vortex rings due to the roll-up of the vortex sheet of the jet are resolved. Computations reveal the same three-dimensional streamwise vortex pairs of jets from circular nozzles observed in experiments by Liepmann and Gharib [3] and New et al. [17]. Instability of the azimuthal vortex rings is found to initiate between the first and the second vortex rings. The opposing flow between the two vortex rings of the outward moving jet induced by the first vortex ring and the inward entrainment of the ambient fluid by the second vortex ring promotes the instability of the vortex rings and thus the generation of the streamwise vortices. The streamwise vortex pairs are generated both inside and outside the vortex rings. As we move further downstream, the azimuthal vortex rings break into segments while the streamwise vortices grow stronger. The streamwise vortices greatly enhance the entrainment and mixing of the ambient fluid with the jet core. The interactions amongst the vortex rings and the streamwise vortices generate complex and fine flow structures downstream leading to the transition to turbulent flow. Although the experiments by Liepmann and Gharib [3] and New et al. [17] show some irregularities in the locations of the streamwise vortex pairs and the mixing patterns of the jet core whereas the computations yield more symmetric locations of the vortices and the mixing patterns of the numerical dye concentration, the basic features of the computed vortex structures of the jet are in good agreement with the experimental observations.

Computed flow structures of the V-nozzle agree very well with the experimental observations by New et al. [17]. The notches of the nozzle lip reduce the small irregularities of the experimental flow pattern of the C-nozzle by Liepmann and Gharib [3]. Computations of the V- and A-notched nozzles show that the basic process and mechanism of the formation of the vortex rings, the generation of the streamwise vortices, and the complex mixing patterns of the jet due to the mutual interaction of the vortex rings and streamwise vortices are the same as those for the regular C-nozzle. However, the notches determine the locations of the initial distortion of the vortex sheet, and thus the mode shape of the instability of the vortex rings and locations where streamwise vortices are generated, leading to different entrainment and mixing patterns of the ambient fluid with the jet core. The sharp corners of the notched lips tend to promote instability of the vortex rings more than the round corners of the lip. The jet spreads faster in the peak-to-peak and trough-to-trough cross-planes than in other directions. Overall the jet from the notched nozzles spreads out faster than that from the regular C-nozzle. The notches in the nozzle exit significantly enhance jet mixing.

4. Conclusions

The vortex flow in the near field of jets from a regular circular nozzle and two notched nozzles are computed by solving the unsteady Navier–Stokes equations. A numerical dye concentration method is used to visualize the flow field and also provide comparison with previous experimental observations. Azimuthal vortex rings due to the roll-up of the vortex sheet of the jet are resolved. Computations reveal the same three-dimensional streamwise vortex pairs of jets from circular nozzles observed in experiments by Liepmann and Gharib [3] and New et al. [17]. Instability of the azimuthal vortex rings is found to initiate between the first and the second vortex rings. The opposing flow between the two vortex rings of the outward moving jet induced by the first vortex ring and the inward entrainment of the ambient fluid by the second vortex ring promotes the instability of the vortex rings and thus the generation of the streamwise vortices. The streamwise vortex pairs are generated both inside and outside the vortex rings. As we move further downstream, the azimuthal vortex rings break into segments while the streamwise vortices grow stronger. The streamwise vortices greatly enhance the entrainment and mixing of the ambient fluid with the jet core. The interactions amongst the vortex rings and the streamwise vortices generate complex and fine flow structures downstream leading to the transition to turbulent flow. Although the experiments by Liepmann and Gharib [3] and New et al. [17] show some irregularities in the locations of the streamwise vortex pairs and the mixing patterns of the jet core whereas the computations yield more symmetric locations of the vortices and the mixing patterns of the numerical dye concentration, the basic features of the computed vortex structures of the jet are in good agreement with the experimental observations.

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References