Multiple Solutions and buffet of Transonic Flow over NACA0012 Airfoil

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Multiple solutions of the small-disturbance potential equation and the full potential equation were known for the NACA0012 airfoil in a certain range of transonic Mach numbers and at zero angle of attack. However, the multiple solutions for this particular airfoil were not observed using Euler or Navier-Stokes equations under the above flow conditions. In the present work, both the Unsteady Reynolds-Averaged Navier-Stokes (URANS) computations and transonic wind tunnel experiments are performed to further study the problem. The results of the two methods reveal that buffet appears under certain Reynolds number and transition position in a narrow Mach number range where the potential flow methods predict multiple solutions. Boundary layer displacement thickness computed from URANS at the same flow condition is used to modify the geometry of the NACA0012 airfoil. Euler equations are then solved for the modified geometry. The results show that the addition of the boundary layer displacement thickness creates multiple solutions for the NACA0012 airfoil. Global linear stability analysis is also performed on the original airfoil and the modified airfoil. This shows a close relationship between the viscous unsteady shock buffet phenomenon of transonic airfoil flow and the existence of multiple solutions of the external inviscid flow.

Nomenclature

- $a$ = speed of sound
- $c$ = chord of airfoil
- $c_p$ = pressure coefficient
- $\bar{c}_p$ = time-averaged pressure coefficient
- $E$ = total internal energy
- $F_c$ = inviscid convective flux
- $F_d$ = inviscid diffusive flux
- $f$ = frequency
- $\bar{f}$ = reduced frequency, $\bar{f} = 2\pi f c/U_\infty$
- $k$ = turbulent kinetic energy
- $l$ = length of tailing edge splitter
- $M$ = Mach number
- $p$ = static pressure
- $t$ = time
- $u, v, w$ = velocity components
- $W$ = conservative variable vector

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I. Introduction

Multiple solutions of the transonic small-disturbance potential equation and the full potential equation have been found for the NACA0012 airfoil under certain conditions of transonic Mach numbers and angles of attack. Jameson reported lifting solutions of the transonic full potential flow equation of the NACA0012 airfoil at zero angle of attack in a narrow band of freestream Mach numbers. Luo et al. showed multiple solutions of transonic small-disturbance equation for NACA0012 airfoil at zero angle of attack within certain range of freestream Mach numbers. However, multiple solutions for the NACA0012 airfoil were not observed using the Euler or the Navier-Stokes equations under the conditions where multiple solutions are found with the potential equation. In order to study the mechanism of multiple solutions, Williams et al. studied stability of multiple solutions of the NACA0012 airfoil with unsteady transonic small-disturbance equation. Liu et al. presented a linear stability analysis of multiple solutions of the transonic small-disturbance potential equation for the NACA0012 airfoil at zero angle of attack. They found the symmetric solutions were not stable and the asymmetric solutions were stable. Crouch et al. demonstrated that the NACA0012 airfoil buffet onset is related to the global stability of the flow field. Liu et al. performed unsteady numerical simulations for the NACA0012 airfoil and presented that the buffet occurs in close range over which multiple solutions happen for the potential equations. It is conjectured that there is a close relationship between shock buffet and the existence of multiple solutions of the external inviscid flow for a transonic airfoil.

In the present work a combined approach of transonic wind tunnel experiments and Unsteady Reynolds-Averaged Navier-Stokes (URANS) computations is used to study the NACA0012 airfoil buffet phenomenon at free stream Mach numbers from 0.82 to 0.89 at zero angle of attack and Reynolds number $3.0 \times 10^6$ based on the chord length. Euler equations are solved for the original NACA0012 airfoil and a modified NACA0012 airfoil by adding the boundary layer displacement thickness computed from URANS to the original NACA0012 airfoil. Finally a global linear stability analysis is performed to study the Euler simulation results.

II. Experimental Details

II.A. Wind Tunnel

The present study was carried out in the continuous closed-circuit transonic wind tunnel of the Northwestern Polytechnical University, Xi’an, China. The two-dimensional test section size is of $0.8 \times 0.4 \times 3 \text{ m}$. The stagnation pressure and the stagnation temperature of the tunnel air were controlled from 0.5 to 5.5 atmospheric pressure and between 283 $K$ and 323 $K$, respectively, dependent on Reynolds number and Mach number. The air was dried until the dewpoint in the test section was reduced sufficiently to avoid condensation effects. The upper and lower walls are 6%-perforated. The holes are $60^\circ$ inclined upstream. The ratio of sidewall-displacement-thickness to tunnel width is about $2\delta^*/b \approx 0.025$. This facility is driven by a two-stage axial-flow compressor.

The flow uniformity in the test section was shown by measuring the centerline static pressure distribution from which the centerline Mach number distribution was calculated using an average total pressure measured
in the still air chamber. Figure 1 gives the Mach number distribution along the centerline of the empty test section of the wind tunnel for nominal Mach numbers from 0.20 to 1.05. The Reynolds number is about $(15 \times 10^6 \text{m}^{-1})$. The flow is uniform in the test section except within 200 mm from the entrance and the exit for Mach numbers between 0.20 and 1.00. In the meantime, the static pressure on the sidewall center line at 200 mm from the entrance was measured. The correlation between the sidewall static pressure and the Mach number in the model region is used to determine the freestream mach number $M_{\infty}$ in the airfoil model testing.

II.B. Model

The model is an NACA 0012 airfoil with a chord length $c = 200$ mm, a span of 400 mm (which gives an aspect ratio of 2.0). The model blockage is 3.0%. The central region of the model is equipped with 81 chordwise static pressure orifices among which 46 and 33 orifices are staggeredly located on the upper and lower surfaces of the airfoil, respectively, and a forward and a rearward facing orifices at the leading and trailing edges, respectively. Besides, there are 10 spanwise static pressure orifices on the upper surface of the airfoil to check two-dimensionality of flow field. The static pressure orifices were 0.3 mm in diameter. The static pressure were measured with electronically actuated differential pressure-scanning-valve units with transducer ranges of $\pm 103 \text{kN/m}^2$, $\pm 206 \text{kN/m}^2$ and $\pm 413 \text{kN/m}^2$ ($\pm 15, \pm 30, \pm 60 \text{lb/in}^2$). Accuracy of the transducers was within 0.05% full scale. The sampling rate is 200 Hz.

Also a limit number of kulite pressure transducers are used to measure dynamic pressure to detect buffet onset. The sampling rate is 20,000 Hz. The dynamic Kulites are mounted close to the upper surface of the airfoil on the support sidewalls at $x/c = 0.25, 0.50$, and 0.75 as shown in Fig. 2. The dynamic pressure orifice diameter was about 2.4 mm.

The model was machined from stainless steel with embedded pressure tubes. The measured coordinates of the experimental model deviate from the coordinates given in Ref. 8 no greater than $\Delta y/c = 0.0002$. The angle of attack is changed manually by rotating the model about pivots of the supports in the tunnel sidewalls. The model with artificial transition strips at $x/c = 5\%$ of chord length is shown in Figure 3.

III. Computational Methods

The computational fluid dynamics code used here is known as PARCAE and solves the unsteady three-dimensional compressible Navier-Stokes equations on structured multiblock grids using a cell-centered finite-volume method with artificial dissipation as proposed by Jameson et al. Information exchange for flow computation on multiblock grids using multiple CPUs is implemented through the MPI (Message Passing Interface) protocol. The Navier-Stokes equations are solved using the eddy viscosity type turbulence models. All computations presented in this work are performed using Menter SST $k-\omega$ model. The main elements of the code are summarized below.

The differential governing equations for the unsteady compressible flow can be expressed as follows:

$$\frac{\partial \textbf{W}}{\partial t} + \nabla \cdot (\textbf{F}_c - \textbf{F}_d) = 0$$

(1)

The vector $\textbf{W}$ contains the conservative variables $(\rho, \rho u, \rho v, \rho w, \rho E)^T$. The fluxes consist of the inviscid convective fluxes $\textbf{F}_c$ and the diffusive fluxes $\textbf{F}_d$, defined as

$$\textbf{F}_c = \begin{pmatrix} \rho u & \rho v & \rho w \\ \rho uu + p & \rho uv & \rho uw \\ \rho vu & \rho vv + p & \rho vw \\ \rho wu & \rho wv & \rho ww + p \\ \rho E'u + pu & \rho E'v + pv & \rho E'w + pw \end{pmatrix}$$

(2)
Mach numbers. Spectral analysis of the individual unsteady pressures at various test cases is made. The mean separated region during buffet leads to a pressure decrease at the trailing edge. Intermittence during shock oscillation and indicates buffet occurrence. Note also that the thickening of the spreading of the recompression region for $M = 3$ indicates that the flow is essentially two-dimensional. The figure also shows the shock waves are much steeper and stronger for $M = 0.86$ and $0.87$, and the shock waves are much weaker for $M = 0.88$ and 0.89. The thickening of the mean separated region during buffet leads to a pressure decrease at the trailing edge.

In order to study buffet phenomenon for the NACA0012 airfoil, the unsteady pressure data on the airfoil are measured from the 6 fast-response Kulite transducers installed on the side walls of wind tunnel. The two foremost transducers ($x/c = 0.25$) are found defective in the present tests. As the flow about the model is two-dimensional, the ensemble-averaged pressure coefficients obtained from the side wall Kulite measurements lie close to the steady pressure distribution measured along the central station of the airfoil.

Figure 4 compares the static pressure measured on the central section of the NACA 0012 airfoil and the time-averaged dynamic pressures measured at the wind tunnel side wall for $M = 0.86, 0.87, 0.88$ and 0.89 at $\alpha = 0^\circ$, $Re = 3.0 \times 10^6$ and $x_T/c = 5\%$. The two different measurements match very well, which indicates that the flow is essentially two dimensional. The figure also shows the shock waves are much steep and stronger for $M = 0.86$ and 0.87 and the shock waves are much weaker for $M = 0.88$ and 0.89. The spreading of the recompression region for $M = 0.88$ and 0.89 results from the temporal integration of the intermittence during shock oscillation and indicates buffet occurrence. Note also that the thickening of the mean separated region during buffet leads to a pressure decrease at the trailing edge.

Figure 5 shows the time histories of dynamic pressure for $M = 0.86 - 0.89$, $\alpha = 0^\circ$, $x/c = 0.75$, at $Re = 3.0 \times 10^6$, $x_T/c = 5\%$, where $x/c$ denotes the location of the Kulite transducer. It also shows there are clearly periodic oscillation of pressure for $M = 0.88$ and 0.89, which confirms buffet occurrence at these Mach numbers. Spectral analysis of the individual unsteady pressures at various test cases is made.

The closure model used to evaluate the turbulent viscosity $\mu_T$ is the $k - \omega$ SST turbulence model, given by the equations

$$
\begin{align*}
\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho ku - \mu_k \nabla k) &= \rho S_k \\
\frac{\partial \rho \omega}{\partial t} + \nabla \cdot (\rho \omega u - \mu_\omega \nabla \omega) &= \rho S_\omega
\end{align*}
$$

where $\mu_k = \mu_L + \sigma_k \mu_T, \mu_\omega = \mu_L + \sigma_\omega \mu_T, \mu_T = \frac{\rho_{\mu k}}{\max(a,\omega,\omega^2) \mu_T}$. The source term $S_k$ and $S_\omega$ are

$S_k = \frac{1}{\rho} \tau : \nabla u - \beta^* \omega k$

$S_\omega = \frac{\gamma}{\mu_T} \tau : \nabla u - \beta \omega^2 + 2(1-f_1) \frac{1}{\omega} \nabla k \cdot \nabla \omega$

In the above equations, $f_1$ and $f_2$ are blending functions. The parameters $\sigma_k, \sigma_\omega, \beta, \beta^*$, and $\gamma$ are closure coefficients.

The flow and turbulence equations are discretized in space by a structured hexahedral grid using a cell-centered finite-volume method. Since within the code each block is considered as a single entity, only flow and turbulence quantities at the block boundaries need to be exchanged. The governing equations are solved using a dual-time stepping method for time accurate solutions. Within each sub-iteration the five stage Runge-Kutta scheme is used with local-time stepping, residual smoothing, and multigrid for convergence acceleration. The turbulence model equations are solved using stagger-couple method. Further details of the numerical method can be found in Ref. 12 and 13.

### IV. Results & Discussion

**IV.A. Experimental Results**

In order to study buffet phenomenon for the NACA0012 airfoil, the unsteady pressure data on the airfoil are measured from the 6 fast-response Kulite transducers installed on the side walls of wind tunnel. The two foremost transducers ($x/c = 0.25$) are found defective in the present tests. As the flow about the model is two-dimensional, the ensemble-averaged pressure coefficients obtained from the side wall Kulite measurements lie close to the steady pressure distribution measured along the central station of the airfoil.

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The spectral analysis of the individual unsteady pressures is done with a discrete Fourier transform. The sampling rate is 20,000Hz and low-pass filtered at 2,000Hz. The sampling length is fixed to 2s. Figure 6 presents the resulting modulus squared of the dynamic pressure coefficient \( |C_p|^2 \) versus frequency \( f \) for \( M_\infty = 0.8 \) and 0.89, \( \alpha = 0^\circ \) and \( x/c = 0.75 \), at \( Re = 3 \times 10^6 \), \( x_{tr}/c = 5\% \). In Fig. 6, a prominent spectral peak with large amplitude near 100 Hz is seen for \( M_\infty = 0.88 \) and 0.89 at \( \alpha = 0^\circ \). No large amplitude peak is found for \( M_\infty = 0.80 \) and 0.87. Table 1 presents the buffet frequency \( f \) and the buffet reduced frequency \( k = 2\pi f c/\infty \) for the NACA 0012 airfoil at \( Re = 3 \times 10^6 \) and \( x_{tr}/c = 5\% \). The dynamic pressures measured at the location \( x/c = 0.50 \) yield no prominent spectral peak and the corresponding spectral figure is not shown for space limitation. More detailed experimental results can be found in Ref. 14.

<table>
<thead>
<tr>
<th>( M_\infty )</th>
<th>( \alpha )</th>
<th>( f ) (Hz)</th>
<th>( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88</td>
<td>0(^\circ)</td>
<td>95.15</td>
<td>0.423</td>
</tr>
<tr>
<td>0.89</td>
<td>0(^\circ)</td>
<td>99.65</td>
<td>0.439</td>
</tr>
</tbody>
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### IV.B. Unsteady Viscous Flow Simulations

In this section the unsteady flow field simulations for the NACA0012 airfoil at freestream Mach numbers from 0.82 to 0.89 at zero angle of attack for Reynolds numbers 3.0 million with transition fixed at 5\% chord length are performed to study unsteady viscous flow behaviors.

The computational results reveal that the flow is steady when the Mach number is between 0.82 and 0.84. When the Mach number is increased to 0.85, the flow suddenly changes into an unsteady oscillatory mode. The shock waves on the upper and lower surfaces of the airfoil begin to move back and forth in a periodic motion. The unsteadiness is caused by the shock wave interaction with the boundary layer over the airfoil surface. It can be categorized to type A shock wave motion.\textsuperscript{15} The unsteady flow pattern persists as the Mach number is further increased to 0.875 where the flow becomes steady again. Figure 7 shows the two extreme shock wave positions against Mach number. In the figure the squares are time-averaged shock wave postion. The reference shock wave position is where the time dependent isentropic Mach number is 1.0. It clearly shows the flow is steady when Mach number is lower than 0.85 or higher than 0.87 and the time-averaged shock wave position moves to trailing edge as the Mach number increases. Buffet occurs in a narrow band of freestream Mach numbers of 0.85-0.87. Figure 8 shows the density gradient contours at 4\% of 18\%.

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As the upper surface shock wave moving upstream, it becomes weaker. At the same time the lower surface shock moves downstream and becomes stronger. Stronger shock induces flow separation. As the separation region on the lower surface becomes large enough, the lower surface shock wave is pushed upstream and the upper surface shock wave moves downstream. The consequence of this behavior is an unsteady shock oscillation over the airfoil.

Figure 9 shows the lift coefficient evolution history for the three Mach numbers of 0.85, 0.86, and 0.87. Figure 10 shows FFT analysis of the lift coefficients. The FFT amplitudes are normalized by the largest peak between the three Mach numbers. Figure 11 shows the comparison of the time-averaged pressure distribution on the NACA0012 surface between computational and experimental data. The time-averaged shock wave positions are very close. Figure 12 shows the comparison of pressure oscillation FFT analysis at \( x/c = 0.75 \). It shows the primary frequencies of pressure oscillation are very close. In the figure the computational Mach number is 0.85 and the experimental Mach number is 0.88. The Mach number discrepancy is caused by the endwall interference of wind tunnel test.

### IV.C. Steady Euler Inviscid Simulations

In this section the steady inviscid flow field simulations for the original and the modified NACA0012 airfoils are performed to study multiple solutions.

The boundary layer displacement thickness computed from URANS in the last section at the Mach number 0.86, at zero angle of attack, and Reynolds number \( 3 \times 10^6 \) is used to modify the geometry of the NACA0012 airfoil. The URANS time-averaged pressure distribution and the boundary layer displacement
thickness are plotted in Fig.13. Before the shock wave the boundary layer displacement thickness is very small. After the shock wave the boundary layer displacement thickness grows significantly, because the shock wave induces flow separation. Figure 14 shows the geometries of the original and the modified NACA0012 airfoils. The rear port of modified airfoil is much thicker than the original NACA0012 airfoil. So the surfaces of the modified airfoil become flat and the tailing edge is not closed.

A C type mesh is generated for the inviscid computations. The mesh size is $257 \times 49$. Figure 15 shows the computational meshes for both airfoils. In order to deal with the open tailing edge problem of modified airfoil, a 'transparent' cusp tailing edge technique is used. Figure 16 shows the converging histories and lift coefficients histories at $M_{\infty} = 0.86, \alpha = 0^\circ$. For original NACA0012 airfoil, the residual converges relative smoothly to machine zero and the lift coefficients are zero during the iterations. It indicates the solution of original NACA0012 airfoil at the flow condition is symmetric. While for the modified NACA0012 airfoil, the residual converges to machine zero. The lift coefficient curve shows the solution is symmetric at the beginning and after the residual jump the solution becomes asymmetric. So there are two solutions for the modified NACA0012 airfoil at the flow condition $M_{\infty} = 0.86, \alpha = 0^\circ$. One is the symmetric solution A, and the other is the asymmetric solution B. Figures 17 – 20 show the Mach number contours and pressure distributions for the original and the modified airfoils at $M_{\infty} = 0.86, \alpha = 0^\circ$. For the original NACA0012 airfoil, the solution is symmetric. The shock wave occurs at 70% of chord length. For the modified airfoil, there are two solution A and B. For the symmetric solution A, the first shock wave occurs at 55% of chord length and followed by small expansion. The second shock wave occurs at 85% of chord length. In the asymmetric solution B, also the first shock wave occurs at 55% of chord length and followed by small expansion. The second shock wave moves to about 80% of chord length on the upper surface. On the lower surface the second shock wave moves to the tailing edge. The asymmetric second shock waves on the upper and lower surface produce non-zero lift of the modified airfoil. Figure 21 shows the lift coefficients versus the angle of attack for the modified NACA0012 airfoil at $M_{\infty} = 0.86$. It shows there are 3 different solutions at zero angle of attack. The different solutions are obtained by using different initial conditions. Figure 22 shows the lift coefficients versus the Mach number for modified NACA0012 airfoil at zero angle of attack. It shows the asymmetric solutions occurs at small range of Mach number $M_{\infty} = 0.85 – 0.86$.

IV.D. Linear stability analysis

In this section the linear stability analysis is performed to analyze stability of the solutions of original and modified NACA0012 airfoils at $M_{\infty} = 0.86, \alpha = 0^\circ$. In the analysis the flow field Jacobian matrix is obtain by perturbing the Euler equation residuals.\(^{16}\) The Euler equations can be expressed as follows:

$$\frac{\partial W}{\partial t} = R(W)$$

The fourth-order accurate Jacobian matrix is

$$\frac{\partial R}{\partial W} = \frac{1}{12\epsilon d}(-R(W + 2\epsilon d) + 8R(W + \epsilon d) - 8R(W - \epsilon d) + R(W - 2\epsilon d))$$

The discretized eigenvalue problem of a Jacobian matrix is solved using the implicit restarted Arnoldi method.\(^{17}\) The solution is achieved by using the ARPACK\(^ {17}\) with the shift-invert mode. The prescribed frequency is set to be zero which is the threshold of unsteadiness. The eigenvalues for the original NACA0012 symmetric solution and the modified NACA0012 airfoil symmetric solution A and asymmetric solution B are examined. For the original NACA0012 airfoil, all the eigenvalues have negative real parts which indicates the solution is stable. The least stable eigenvalue has a real part of $-0.0097$. For the modified NACA0012 airfoil symmetric solution A, the eigenvalue largest real part is positive 0.015. It means the symmetric solution A is unstable. For the modified NACA0012 airfoil asymmetric solution B, all the eigenvalues have negative real parts, which indicates the solution is stable. The least stable eigenvalue has a real part of $-0.0092$. Figure 23 shows the pressure fluctuation eigenmodes corresponding to the least stable or unstable eigenvalue. For the original NACA0012 airfoil, the flow field is symmetric and the pressure fluctuation eigenmode is also symmetric. For the modified NACA0012 airfoil solution A, the flow field is symmetric, while the pressure fluctuation eigenmode is asymmetric. For solution B, the flow field and the pressure fluctuation eigenmode are both asymmetric.
V. Conclusion

A combined experimental and computational study is performed to study buffet of the NACA0012 airfoil and its relation to inviscid multiple solutions. The experiments and computations both reveal that buffet appears under certain Reynolds number and transition position in a narrow Mach number range.

Boundary layer displacement thickness computed from URANS at the same flow condition is used to modify the geometry of the NACA0012 airfoil. Euler equations are solved for the original and modified airfoils. For the original NACA0012 airfoil, there are no multiple solutions. While for the modified NACA0012 airfoil, multiple solutions exist. The addition of the boundary layer displacement thickness creates multiple solutions for the NACA0012 airfoil. Global linear stability analysis is performed on the original airfoil and the modified airfoil. This shows a close relationship between the viscous unsteady shock buffet phenomenon of transonic airfoil flow and the existence of multiple solutions of the external inviscid flow. Buffet seems to appear when the external inviscid flow may exhibit multiple solutions and the mean flow field exhibit global instability.

References

Figure 1. Mach number distribution along the centerline of the empty test section of the wind tunnel.

Figure 2. Location of dynamic pressure measurements orifices on the support sidewall.

Figure 3. Model in the wind tunnel
Figure 4. Experimental mean surface pressure coefficient, NACA 0012, $M_{\infty} = 0.86 - 0.89$, $\alpha = 0.0^\circ$, $Re = 3.0 \times 10^6$, $x_{tr}/c = 5\%$.

Figure 5. Experimental dynamic pressure coefficients at $x/c = 0.75$, NACA 0012, $M_{\infty} = 0.86 - 0.89$, $\alpha = 0.0^\circ$, $Re = 3.0 \times 10^6$, $x_{tr}/c = 5\%$. 

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Figure 6. Experimental dynamic pressure coefficient modulus squared $|\tilde{C}_p|^2$ vs. frequency $f$, NACA 0012, $M_\infty = 0.80 \text{ to } 0.89$, $\alpha = 0^\circ$, $x/c = 0.75$, $Re = 3.0 \times 10^6$, $x_{tr}/c = 5\%$.

Figure 7. URANS computational shock wave position against freestream Mach number for NACA0012 airfoil, $\alpha = 0^\circ$, $Re_\infty = 3.0 \times 10^6$, $x_{tr}/c = 5\%$. (Blue square denotes time-averaged shock wave position)
Figure 8. URANS computational density gradients contours at different instant time in one period around the NACA0012 airfoil, $M_\infty = 0.86, \alpha = 0.0^\circ, Re_\infty = 3 \times 10^6, x_{tr}/c = 5\%$. (a) $t = 1/4T$; (b) $t = 2/4T$; (c) $t = 3/4T$; (d) $t = 4/4T$.

Figure 9. Evolution of URANS computational lift coefficients for NACA0102 airfoil, $\alpha = 0.0^\circ, Re_\infty = 3.0 \times 10^6, x_{tr}/c = 5\%$. 
Figure 10. FFT analysis of URANS computational lift coefficients for NACA0102 airfoil, \( \alpha = 0.0^\circ, Re_\infty = 3.0 \times 10^6, x_{tr}/c = 5\% \).

Figure 11. Comparison of experimental and URANS computational time-averaged pressure distributions around NACA0012 airfoil, \( \alpha = 0.0^\circ, Re_\infty = 3.0 \times 10^6, x_{tr}/c = 5\% \).
Figure 12. FFT analysis of experimental and URANS computational pressure oscillations at \( x/c = 0.75 \) for NACA0012 airfoil, \( \alpha = 0.0^\circ, Re_{\infty} = 3.0 \times 10^6, x_{tr}/c = 5\% \)

Figure 13. URANS computational time-averaged pressure distribution and boundary-layer displacement for NACA0012 airfoil, \( \alpha = 0.0^\circ, Re_{\infty} = 3.0 \times 10^6, x_{tr}/c = 5\% \)

Figure 14. Comparison of geometry between original and modified NACA0012 airfoils.
Figure 15. Computational Meshes (256 × 48). (a) Original NACA0012 airfoil; (b) Modified NACA0012 airfoil.

Figure 16. Euler computational converge history $M_\infty = 0.86, \alpha = 0.0^\circ$. (a) Original NACA0012 airfoil; (b) Modified NACA0012 airfoil.
Figure 17. Euler computational Mach number contours around the original NACA0012 airfoil, $M_\infty = 0.86, \alpha = 0^\circ$. (a) Overall view; (b) Close up;

Figure 18. Euler computational Mach number contours for the symmetric solution A around the modified NACA0012 airfoil, $M_\infty = 0.86, \alpha = 0^\circ$. (a) Overall view; (b) Close up;
Figure 19. Euler computational Mach number contours for the symmetric solution B around the modified NACA0012 airfoil, $M_\infty = 0.86, \alpha = 0.0^\circ$. (a) Overall view; (b) Close up.

Figure 20. Comparison of Euler computational pressure distributions. $M_\infty = 0.86, \alpha = 0.0^\circ$
Figure 21. Euler computational lift coefficients versus angle of attack $\alpha$ of modified NACA0012 airfoil. $M_\infty = 0.86$

Figure 22. Euler computational lift coefficients versus Mach number of modified NACA0012 airfoil. $\alpha = 0.0^\circ$
Figure 23. Euler computational pressure fluctuation eigenmode corresponding to the least stable or unstable eigenvalue, $M_{\infty} = 0.86, \alpha = 0.0^\circ$. (a) Original NACA0012 airfoil; (b) Modified NACA0012 airfoil symmetric solution A; (c) Modified NACA0012 airfoil asymmetric solution B;