Development of Flow Asymmetry over a 20-degree Circular Cone at Low Speed

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This paper reports results from comprehensive pressure tests on a circular cone-cylinder model in a low-turbulence 3.0 × 1.6 m low-speed wind tunnel. The semi-apex angle of the cone is 10°. The results consist of detailed pressure distributions over nine stations along the cone at angles of attack of 0 ~ 35°, and Reynolds number of 0.3 × 10⁶ based on the cone base diameter. The tests encompassed a complete coverage of roll orientations in 9° intervals. The local and overall forces and moments are calculated from the measured pressures. As angle of attack is increased, the variation of the local side force coefficient normalized by the local diameter, with roll angle develops into three stages: zero curve, continuous-wave curve, and square-wave curve. The primary state of the boundary layer is inferred from the measured pressures. The mechanism for pressure asymmetry is identified as the hydrodynamic instability of the separated vortices. The conicity of the flow is examined. Asymmetric features which are common between models of the same geometry and flow specifications are clarified and validated with known theoretical predictions and other experimental observations in the literature. The complete measured pressure data are documented in Appendix for public use.

Nomenclature

\[ C_m \] = pitching-moment coefficient about cone base, pitching moment/\( q_\infty SD \)
\[ C_n \] = yawing-moment coefficient about cone base, yawing moment/\( q_\infty SD \)
\[ C_{Nd} \] = local normal force coefficient, local normal force/\( q_\infty d \)
\[ C_{N0} \] = overall normal force coefficient, overall normal force/\( q_\infty S \)
\[ C_{Yd} \] = local side force coefficient, local side force/\( q_\infty d \)
\[ C_{Y0} \] = overall side force coefficient, overall side force/\( q_\infty S \)
\[ c_p \] = pressure coefficient
\[ D \] = base diameter of circular cone
\[ d \] = local diameter of circular cone
\[ L \] = length of circular cone
\[ M \] = free-stream Mach number
\[ q_\infty \] = free-stream dynamic pressure

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**I. Introduction**

The high angle of attack aerodynamics of a symmetric body under symmetric flight conditions is a problem of both academic interest and practical significance because the symmetric body can produce an asymmetric flow and hence experience a side force which directly affects the maneuverability of an aircraft or missile. A great deal of experimental, theoretical, and computational effort has been spent regarding the understanding, prediction, and control of the vortex asymmetry. The subject has been reviewed by Hunt,1 Ericsson and Reding,2 and Champigny.3

The basic physical mechanism of the force asymmetry is not clear. At least two possible causes for the force asymmetry were suggested mainly based on experimental investigations: (1) Inviscid hydrodynamic instability of the symmetrically separated vortices (Keener and Chapman)4 and (2) asymmetric flow separation and/or asymmetric flow re-attachment on each side of the body (Ericsson).5 There is at present no general agreement on the mechanism involved in the creation of the force asymmetry.

There seems little doubt that the apex of the pointed slender body plays a decisive role in determining the flow pattern over the entire body. Since the pointed nose is locally conical in shape, the flow may be regarded as locally equivalent to that about a tangent cone. The basic features of the asymmetric flow about pointed slender bodies of revolution can be displayed by studying the flows over a circular cone.

Force tests for a circular cone of 8.13° semi-apex angle were conducted by Coe et al.6 for angles of attack up to 75° in two low speed wind tunnel by a strain gage balance system. Asymmetric force and moment onset at about 16° angle of attack at zero sideslip and reached a high magnitude at 35° angle of attack and reversed in direction beyond the angle of attack. The yawing moment coefficient was given for four roll orientations in 90° intervals at a Reynolds number of 0.21 × 10^6 based on base diameter. Similar data were obtained for roll angles of 0°, 180°, and 270°, but a roll angle of 90° produced a reversal in the sense of the asymmetry, the magnitudes of the asymmetry and the angle of attack at which they occurred remained relatively unchanged. The large forces and moments were caused by asymmetric shedding of vortex sheets off the pointed nose as visualized by tuft and smoke techniques. The results were found to be repeatable with no noticeable hysteresis effect.

Keener et al.7 measured the forces and moments acting on a cone of semi-apex angle 10° at Reynolds numbers based on base diameter ranging from 0.3 × 10^6 to 4.6 × 10^6 and Mach numbers ranging from 0.1 to 0.7 by a force balance in a large subsonic wind tunnel. Angle of attack was varied from 0° to 88° at zero sideslip. The cone with pointed nose experienced steady side force at high angles of attack at zero sideslip. A large side force developed at angles of attack between 20° and 75°. The side force changed from side to side as the angle of attack increased and was accompanied by dynamic oscillations. The direction and magnitude of the side force was sensitive to the body geometry near the nose. The maximum side force reached about 0.9 times the normal force. The angle of attack of onset of side force was about 20° and not strongly influenced by Reynolds number or Mach number.

Lowson et al.8 reported the flow asymmetry development over circular cone as function of angle of attack, semi-apex angle and wind speed by smoke-laser-sheet visualization. The cone models had semi-apex angle of 5°, 10° and 20°. The flow over the 5° cone could be seen to be essentially conical at high angles of attack. For the 20° cone, the separated flow was found to be non-conical at lower angles of attack.

An RAE unpublished force measurement by A.R.G. Mundell (1982) was reported in Ref. 9. A large side force was obtained for a 20° cone at a high angle of attack, zero sideslip and low speed by a balance, while the separation positions were not greatly asymmetric as recorded by oil flow method.

Fiddes9 reported that a pressure measurement of a nose cone-fairing segment-after cylinder model had been conducted in the RAE 5 m low-speed pressurized wind tunnel. The nose cone has 10° semi-apex angle.
The model was rolled in 10° intervals through the entire roll range. There were six pressure-tapping stations in which four stations were located on the nose cone and the rest two stations on the fairing segment. Each station housed 36 holes equally-spaced around the circumference. Part of the data on the front station, 148.5 mm from the apex at 35° angle of attack was reproduced in Ref. 9.

The slender conical flow of an incompressible inviscid fluid has been analyzed by many authors. Dyer et al. found non-unique stationary (symmetric and asymmetric) vortex-pair positions even when symmetric separation positions are postulated with respect to the incidence plane. The stability of the stationary solutions were studied by Pidd et al. and Cai et al. Pidd et al. showed that the stationary conical solutions are convectively stable under certain conditions. Cai et al. found that no stationary conical solutions are stable based on the global stability analysis.

The purpose of the present paper is to study how, when, and why asymmetry occurs over a slender circular cone at high angles of attack and zero sideslip. The work starts with extensive pressure measurement, since no complete measurement data are available to the present authors. Flow field surveys were not made in the present tests.

In the following sections, the experimental setup is presented and verified at zero angle of attack. The variation of the local side force and normal force with roll angle at nine longitudinal stations are calculated. The primary boundary-layer state is inferred from the measured pressures. The mechanism for asymmetry generation is examined. The conicity of the flow is studied. Asymmetric features independent of roll angle are clarified and validated with known theoretical predictions and other experimental observations. Conclusions are lastly offered. The pressure-measurement data are documented in the Appendix for public use.

II. Experimental Setup

All the tests of this paper are conducted in the NF-3 wind tunnel at the Aerodynamic Design and Research National Laboratory, Northwestern Polytechnical University. The test section has a 3.0 x 1.6 m cross section, and a length of 8.0 m. The contraction ratio is 20 : 1. The free-stream turbulence level is 0.045% for wind speeds of 20 ~ 130 m/s.

The model comprises a nose cone of 10° semi-apex angle faired to a cylindrical afterbody as shown in Fig. 1. The longitudinal distance from the rear pressure-measurement station to the front support axle is about twice as large as the measurement length. No flow distortion is detected over the pressure-measurement stations by the presence of the support axle.

The tip portion of the cone from the apex to a station of 150.0 mm along the body axis is separately made to facilitate the pressure-tube installation inside the model. After assembling the two portions, the junction was filled with grease and polished. The model is made of metal and constructed to an average tolerances of ±0.05 mm with a surface finish of nearly ±0.8 μm. The fore-body including the nose cone and the fairing portion is roll-able and the after cylinder is mounted onto the model support. The junction between the fore-body and the after cylinder is carefully machined so that the surface discontinuity is less than 0.025 mm.

The fore-body’s roll orientation is facilitated by clamping the fore-body to the axis of a remotely controlled motor which is housed in the after cylinder. The model can be set at any roll angle between 0 and 351° from the chosen datum in 9° intervals. The accuracy of the roll-angle set is about 1%. The pressure instrumentation is confined to the nose cone and is well forward of the model support. The pressure tapings are placed at 9 stations along the model’s axis as shown in Fig. 1. Stations 1 and 2 have 12 and 18 pressure orifices, respectively, and the rest stations have 36 pressure orifices. The pressure orifices in each station are equally-spaced around the circumference and arranged from the same datum for all stations. The total number of the pressure orifices is 282.

The static pressure at each pressure orifice is transmitted by a rubber tube passing through the base of the after cylinder to the pressure-measurement system outside the test section. The system consists of 24 scan-valves each of which has 16 channels and one pressure transducer of modulus 9816 of the PSI Company with an accuracy of ±0.05%. The pressure-measurement readings for each test case were taken 115 times in 0.05 s intervals and then time-averaged. The fluctuations of the readings are small and the time-averaged data are meaningful. A thorough job of cleaning the model was done prior to each run of the wind tunnel.

Figure 2 shows the model rigidly supported in the wind tunnel.
Figure 1. The cone-cylinder model.
Figure 2. The model in the wind tunnel.
The cone-cylinder model is tested at \( \alpha = 0^\circ \sim 35^\circ \), \( V = 30 \, m/s \), \( M = 0.09 \) and \( Re = 0.3 \times 10^6 \). 40 roll angles in 9\(^\circ\) intervals are tested. Readings from 12 out of the 282 pressure orifices are abnormal in the tests. The abnormality may be caused by the twisting of the bunch of the 282 pressure tubes during the fore-body roll. The corrected pressure coefficient is calculated with a linear interpolation from the neighboring normal values. There are no abnormal pressure readings at Stations 1,3,8,9. Table 1 gives the pressure orifice number which yields abnormal pressure readings on various stations. The pressure orifice is numbered in the counter clockwise direction with Orifice 1 located at \( \theta = 230^\circ \) when \( \phi = 0 \) for all measured stations.

Table 1. Number of the pressure orifice which yields abnormal readings on various stations

<table>
<thead>
<tr>
<th>Station</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orifice number</td>
<td>12,14</td>
<td>4,30</td>
<td>1, 6, 10, 11</td>
<td>29,32</td>
<td>3,27</td>
</tr>
</tbody>
</table>

The experimental setup is verified by measured pressure at zero angle of attack. Figure 3 presents the pressure coefficient \( c_p \) versus \( \theta \) at \( \alpha = 0^\circ \) and \( \phi = 0 \) for all stations. The results at other roll angles are similar. (See the Appendix.) The measured pressure is nearly invariant versus meridian angles on each station and decreases monotonically from about 0.1 at Station 1 to about 0.01 at Station 9. This verifies that the experiment setup yields an essentially axi-symmetric flow around the model at zero angle of attack and zero sideslip. The small deviations of \( c_p(\theta) \) from constant, especially on the two front stations, may be attributed to the model-surface imperfections which become significant relative to the local diameter there. While great care was taken to fabricate the model and to set up the test rig, like all experiments, the test rig can never be perfect and small imperfections are unavoidable. This explanation applies to similar observations below.

Verifications with available test data in the literature are given in Section VI.

III. Local Side- and Normal-Force Coefficients versus Roll Angle at Various Angles of Attack

The measured pressures are integrated to give force and moment components in the body coordinate system, \( ox'y'z' \) shown in Fig. 4, where the coordinate plane \( ox'z' \) coincides with the incidence plane. The local side- and normal-force coefficients, \( C_{Y_d} \) and \( C_{N_d} \) are normalized by the local diameter, \( d \) of the cone rather than the base diameter, \( D \) in order to convey more information as shown by Hall.\(^{15}\)

The development of the local side- and normal-force variation with roll angle as function of angle of attack from \( 0^\circ \) upto \( 35^\circ \) is investigated in this section. Figures 5-11 present the local side- and normal-force coefficients, \( C_{Y_d} \) and \( C_{N_d} \) versus roll angle \( \phi \) for odd-numbered stations at \( \alpha = 0^\circ \sim 35^\circ \). The even-numbered stations have similar patterns.
Figure 4. Body axes for the circular cone at an angle of attack.

Figure 5. Local side- and normal-force coefficients vs. $\phi$ at odd-numbered stations, $\alpha = 0^\circ$. 
(a) local side-force coefficient
(b) local normal-force coefficient

Figure 6. Local side- and normal-force coefficients vs. $\phi$ at odd-numbered stations, $\alpha = 10^\circ$.

(a) local side-force coefficient
(b) local normal-force coefficient

Figure 7. Local side- and normal-force coefficients vs. $\phi$ at odd-numbered stations, $\alpha = 15^\circ$.

(a) local side-force coefficient
(b) local normal-force coefficient

Figure 8. Local side- and normal-force coefficient vs. $\phi$ at odd-numbered stations, $\alpha = 20^\circ$. 
Figure 9. Local side- and normal-force coefficients vs. $\phi$ at odd-numbered stations, $\alpha = 25^\circ$.

Figure 10. Local side- and normal-force coefficients vs. $\phi$ at odd-numbered stations, $\alpha = 30^\circ$.

Figure 11. Local side- and normal-force coefficients vs. $\phi$ at odd-numbered stations, $\alpha = 35^\circ$. 
At $\alpha = 0$, the local side- and normal-force coefficients are indeed very small: $C_{Yd} \approx 0.005$ and $C_{Nd} \approx -0.01$.

The development of the flow asymmetry as function of angle of attack can be classified into three stages according to the variation pattern of the curves, $C_{Yd}(\phi)$ and $C_{Nd}(\phi)$.

Stage 1. $10^\circ < \alpha < 20^\circ$: The local side-force coefficient remains a small magnitude about $-0.02$, but the local normal-force coefficient grows up to $0.15$ at $\alpha = 10^\circ$, and $0.35$ at $\alpha = 20^\circ$. It indicates that symmetric flow prevails in this range of angle of attack. A symmetric separation onset at $\alpha \approx 10^\circ$ is seen from the appearance of non-linear variation of the limiting values of the overall normal-force over all roll angles versus $\alpha$ in Fig. 25 of Section VI.

Stage 2. $20^\circ < \alpha < 35^\circ$: $C_{Yd}(\phi)$ becomes a continuous-wave curve. The maximum $C_{Yd}$ grows up to about $\pm 0.4$ and $C_{Nd}$ to $0.6$ at $\alpha = 30^\circ$. It indicates that asymmetric separation appears in this range of angle of attack. And the asymmetry has an infinite number of stable states, including a symmetric one.

Stage 3. $\alpha = 35^\circ$: $C_{Yd}(\phi)$ becomes nearly a square-wave curve at this angle of attack. The local side force coefficient takes either maximum positive or maximum negative value and only a few intermediate values appear. The asymmetry has essentially two stable states, i.e., bistable state. The maximum $C_{Yd} \approx \pm 0.6$. $C_{Nd} \approx 0.7$.

In Stages 2 and 3, the initial asymmetry manifests a continuous-wave variation of side force versus roll angle, and with further increase in angle of attack to $35^\circ$ the asymmetry develops into a nearly-square-wave variation. The direction of the local side force at a given roll angle is about the same for all measured stations and at all angles of attack investigated. The change of the pressure asymmetry is triggered by the micro surface imperfections near the cone apex. The boundary layer there is so thin and can be penetrated by the micro surface imperfections of the body. In Stage 2, the asymmetry responds to all local levels of perturbation at each azimuthal angle around the body. In Stage 3, only at certain roll angles the geometric perturbations are effective to the asymmetry change.

The particular pattern obtained for this model can have no general significance, because it pertains to the particular micro surface imperfections of the model. The micro surface imperfections are random and unpredictable. Another model made to the same geometry specifications would have different surface imperfections and, hence, a different roll-angle dependence. However, the development stages of pressure asymmetries as function of angle of attack and the consistency of the asymmetry variation with roll angle for all pressure stations and all angles of attack investigated may be applied generally to cones of the same apex angle under the same flow conditions.

The magnitudes of the maximum positive and maximum negative local side-force coefficients for each station at a given angle of attack are nearly equal. The small deviation may be attributed to micro asymmetries in the experimental setup. Every positive local side force is associated with a negative one with the same magnitude at different roll angles.

The local normal force is nearly invariant with roll angle. At $\alpha = 35^\circ$, the maximum local side force has the same order of magnitude as that of the local normal force. The present pressure measurement shows that an increase of the angle of attack to the apex angle of the cone gives rise to the appearance of initial asymmetry, which agrees with the force measurements by Coe et al.\textsuperscript{6} and Keener et al.\textsuperscript{7}

IV. Primary State of Boundary-Layer from Pressures

Although no flow field surveys were made in the present tests, the primary state of the boundary layer can be inferred from the measured pressures as shown by Hall\textsuperscript{16} by comparing the flow field visualizations with the corresponding pressure measurements. A usual pressure distribution is dense enough to yield the primary picture of the cross flow, but not dense enough to resolve the fine (secondary) structure of the flow. The primary state of the boundary layer, in general, can illuminate the origin of the pressure asymmetry.

For Stage 1, Fig. 12 presents a typical pressure distribution $c_p$ versus meridian angle $\Theta$, where $\Theta$ is $\theta$ and $-\theta$ for the port and starboard side, respectively, and the surface streamline in the cross-flow plane at $\alpha = 15^\circ$, $\phi = 36^\circ$, and Station 3.

The Reynolds number based on $D$ is $0.3 \times 10^6$. At this Reynolds number, the separation may be either laminar separation or transitional separation.\textsuperscript{17} Figure 12 pertains to the laminar separation. The characteristic points of the boundary layer are labeled on the curve of $c_p(\theta)$.

1. Point $A$, attachment point slightly offset to starboard side from incidence plane:
Figure 12. Pressure coefficient vs. $\theta$ and surface streamline in cross-flow plane at $\alpha = 15^\circ$, $\phi = 36^\circ$, Station 3.

2. Point $P_1$, suction peak point on port surface, $\theta = 96^\circ$;
3. Point $S_1$, laminar separation point on port surface, $\theta = 116^\circ$;
4. Point $P_2$, suction peak point on starboard surface, $\theta = -114^\circ$;
5. Point $S_2$, laminar separation point on starboard surface, $\theta = -124^\circ$.

The attachment point $A$ is offset to the starboard side and the boundary layer over the port side is separated earlier than that on the starboard side by about $10^\circ$. They indicate that the model is slightly sideslip to the starboard direction and a small local side force pointing to the port side is resulted in. $C_{Yd} = -0.02$ from Fig. 7.

For Stage 2, Fig. 13 presents the pressure coefficient $c_p$ versus meridian angle $\Theta$ and the surface streamline in the cross-flow plane at $\alpha = 25^\circ$, $\phi = 36^\circ$, and Station 3.

Figure 13. Pressure coefficient vs. $\theta$ and surface streamline in cross-flow plane at $\alpha = 25^\circ$, $\phi = 36^\circ$, Station 3.
The pressure distribution on Fig. 13 shows evidence of a laminar bubble separation followed by a turbulent re-attachment, with a final turbulent separation, i.e., a transitional separation. The characteristic points of the boundary layer are labeled on the curve of $c_p(\theta)$.

1. Point $A$, attachment point slightly offset to starboard side from incidence plane;
2. Point $P_{11}$, first suction peak point on port surface, $\theta = 86^\circ$;
3. Point $S_{11}$, laminar bubble separation point on port surface, $\theta = 96^\circ$;
4. Point $R_1$, turbulent re-attachment point on port surface, $\theta = 126^\circ$;
5. Point $S_{12}$, turbulent separation point on port surface, $\theta = 136^\circ$;
6. Point $P_{12}$, second suction peak point on port surface, $\theta = 156^\circ$;
7. Point $P_{21}$, first suction peak point on starboard surface, $\theta = -94^\circ$;
8. Point $S_{21}$, laminar bubble separation point on starboard surface, $\theta = -114^\circ$;
9. Point $R_2$, turbulent re-attachment point on starboard surface, $\theta = -144^\circ$;
10. Point $S_{22}$, turbulent separation point on starboard surface, $\theta = -154^\circ$;
11. Point $P_{22}$, second suction peak point on starboard surface, $\theta = -164^\circ$;

For Stage 2, there appears a second suction peak after the first suction peak on each side of the body. The first suction peak is naturally formed due to the convex shape of the circular cross-section on each side. The second suction peak is induced by the low pressure of the vortex cores formed from the separation of the boundary layer over each side of the cone. The second suction peak on the starboard side is higher than that on the port side. It indicates that the separation vortex core on the starboard side lies closer to the body surface than that on the port side and thus, the asymmetric vortex pair induces a local side force pointing to the starboard side of the cross section. $C_{Y_{d}} = 0.02$ from Fig. 9.

The first suction peaks make direct contribution to the local side force, and the second suction peaks make direct contribution to the local normal force rather than the local side force. However, the second suction peaks affect the overall cross-flow: the separation point $S_{22}$ on the starboard side lies closer to the leeward central generator than that on the port side $S_{12}$, the separation point $S_{21}$ on the starboard side is delayed, and the first suction peak $P_{21}$ on the starboard side is raised. It is seen that the asymmetric separation and the local side force both are attributed to the asymmetric vortex configuration. The attachment point $A$ is offset to the starboard side as in Stage 1 due to the micro asymmetry in the experimental setup.

The vortex configuration is determined by the interaction between the separated vortices and the boundary conditions on the cone surface. Figure 9 indicates that there are an infinite number of possible vortex configurations in Stage 2 including a symmetric one. The asymmetric vortex configurations appear as mirror images since the cone is axi-symmetric and inclined at zero sideslip. Thus, every positive $C_{Y_{d}}$ is associated with a negative one of the same magnitude at different roll angles. The stable vortex configuration occurs with micro surface imperfections at the body tip acting as an instability trigger. At the body tip, the boundary layer is thin so that micro surface imperfections can penetrate it to affect the flow.

Moreover, beneath the primary vortex described above there almost always appears a secondary separation vortex over the cross section of the model as observed by Keener \cite{18} using an oil-flow technique. To describe the effects of the secondary separation vortex on the pressure distribution, denser pressure data in the corresponding region are required.

For Stage 3, Fig. 14 presents the pressure coefficient $c_p$ versus meridian angle $\Theta$ and the surface streamline in the cross-flow plane at $\alpha = 35^\circ$, $\phi = 36^\circ$, and Station 3.

Figure 14 shows that the boundary layer on the port side pertains to laminar separation and that on the starboard side pertains to transitional separation. The characteristic points of the boundary layer are labeled on the curve of $c_p(\theta)$.

1. Point $A$, attachment point slightly offset to port side from incidence plane;
2. Point $P_1$, suction peak point on port surface, $\theta = 76^\circ$;
3. Point \( S_1 \), laminar separation point on port surface, \( \theta = 96^\circ \);

4. Point \( P_{21} \), first suction peak point on starboard surface, \( \theta = -94^\circ \);

5. Point \( S_{21} \), laminar bubble separation point on starboard surface, \( \theta = -114^\circ \);

6. Point \( R_2 \), turbulent re-attachment point on starboard surface, \( \theta = -134^\circ \);

7. Point \( S_{22} \), turbulent separation point on starboard surface, \( \theta = -154^\circ \);

8. Point \( P_{22} \), second suction peak point on starboard surface, \( \theta = -174^\circ \);

In Stage 3, only one second-suction-peak occurs in the measured pressure, i.e., the second suction peak \( P_{22} \) on the starboard surface. The starboard vortex core lies close to the body surface. No second-suction-peak can be detected on the port side, as the vortex core is too remote from the body. The asymmetry of the vortex pair is enhanced and affects the entire flow. The attachment point \( A \) is offset to the port side of the incidence plane. A large lateral pressure difference is developed, which results in a large positive side force. \( C_{Yd} = 0.52 \) from Fig. 11.

The square-wave variation of the local side force versus roll angle (Fig. 11) indicates there exist two, and only two possible vortex configurations which are mirror images. The separation vortex pair stays with one asymmetric configuration until triggered to its mirror image back and forth by dominant micro surface imperfections at some roll angles. How the micro surface imperfections act as an instability trigger is still unclear.

Lowson et al.\(^8\) reported the process of flow development as function of angle of attack from flow visualization. At low enough angles of attack no separation is visible. As angle of attack is increased symmetric separation appears. For the cone of semi-apex angle 5\(^\circ\) the shear layers feed a pair of vortex cores in the form of the classic tightly rolled vortex. But for the cone of 20\(^\circ\) semi-apex angle a rather different process was observed before the appearance of initial asymmetry.

1. The first effect is the appearance of an extended shear layer close to the body without rolling into the concentrated vortex remote from the body.

2. Further increase in angle of attack causes the extended shear layer to collect into the concentrated vortex. The separation remains symmetric but the flow is clearly non-conical.

The present pressure measurements for the cone of 10\(^\circ\) semi-apex angle match with the flow visualizations\(^8\) cited above fairly well. The present tests show that separation appears at \( \alpha = 10^\circ \) and the separation is
symmetric. At $\alpha = 10^\circ$ and $15^\circ$ the pressure distribution over the cross section of the cone remains symmetric and no second suction peak (i.e., nearby vortex core) is detected. At $\alpha = 20^\circ$ a pair of second suction peaks appears and the pressure distribution remains symmetric. At $\alpha = 25^\circ$ the second suction peaks become asymmetric, and the pressure distributions become asymmetric. At $\alpha = 30^\circ$ and $35^\circ$ only one second suction peak is detected and the pressure asymmetry becomes strong.

The development of the pressure distribution holds true for other roll angles and other stations. (See details in the Appendix.) The analysis of the pressure distribution in this section clearly shows that the pressure asymmetry is induced by the asymmetric vortex configuration. The vortex configuration results from the interaction between the separation vortices and the boundary conditions on the body. The interaction may yield multiple mirror-imaged results, including a symmetric one. The stable one occurs with microasymmetries in the geometry of the body tip acting as the instability trigger. The continuous side-force variation with roll angle responds to the local levels of the surface imperfections at each azimuthal angle around the body. The nearly square side-force variation is established by the dominant features of the surface imperfections. The boundary conditions on the model are independent of the micro surface imperfections of the model, and so are the vortex configurations. The mechanism for pressure asymmetry is identified as the hydrodynamic instability of the separated vortices. How the micro surface imperfections actually act as an instability trigger remains unclear.

V. Pressure and Side- and Normal-Force Distributions along the Cone Axis

The distributions of the measured pressure along meridian lines and the distributions of the integrated local side- and normal-force coefficients along the cone axis are studied.

Figure 15 presents the pressure coefficients on six meridian lines in $60^\circ$ intervals versus longitudinal coordinate, $x/L$ at $\phi = 171^\circ$, and at $\alpha = 10^\circ - 35^\circ$. The results at other roll angles are similar. The pressures measured on a given meridian line are not constant along the cone axis.

The distributions of the local side- and normal-force coefficients along the cone axis are presented in Figs. 16 and 17 at $\alpha = 30^\circ$ and $35^\circ$, respectively, and at four roll angles in intervals of $90^\circ$. Similar to the pressure distribution, the local side- and normal-force coefficients at a given roll angle are not constant along the cone axis especially in the tip region. The large scattering at Station 2 may be caused by the large local surface imperfections there. It is noted that $C_{Y_d}$ would approach a non-zero value when $x$ approaches zero. Since $C_{Y_d}$ is normalized by the local diameter $d$, the sectional side force is proportional to $d$ as the section approaches the body apex. Therefore, the pressure asymmetry does occur right at the apex of the body and evolved down the body.

While the onset of wavy variation of the local side force versus roll angle appears at $\alpha = 20^\circ$, the onset of non-conicity occurs at $\alpha = 10^\circ$. The non-conicity onset validates the theoretical prediction that no conical vortex flows (symmetric or asymmetric) over a slender circular cone at high angles of attack are stable under small perturbations, no matter the separation position is postulated symmetric or asymmetric.\textsuperscript{13,14} The present results are confirmed by other experimental observations. Pidd et al.\textsuperscript{12} reported non-conical flows in tests of a circular cone of semi-apex angle of $10^\circ$ at $\alpha = 35^\circ$ and many roll angles in an RAE 5 m low-speed wind tunnel. Lowson et al.\textsuperscript{8} observed that the flow is non-conical before the appearance of initial asymmetries for a cone of $20^\circ$ semi-apex angle as the angle of attack is increased.

VI. Overall Forces and Moments versus Roll Angle and Verifications

The overall forces and moments are calculated from integrating the measured pressure at the nine stations. The longitudinal length of the cone from the apex to the base is divided into nine elements by the eight mid points between the neighboring stations. On each element, the pressure distributions are assumed to be constant along same meridian line. The overall normal- and side-force coefficient, $C_{N_0}$ and $C_{S_0}$ are defined by the base area of the cone $S$. The overall pitching- and yawing-moment coefficients, $C_m$ and $C_n$ are defined by $S$ and the base diameter, $D$. The moment center is positioned at the base of the cone.

The overall force and moment coefficients versus roll angle at various angles of attack are presented in Figs. 18 to 24. The development of the flow as function of angle of attack is again obtained. All four components are nearly zero at $\alpha = 0$. As $\alpha$ is increased to $10^\circ$ the two normal components become non-zero but the two lateral components remain zero until $\alpha = 20^\circ$. The flow has a separation but is symmetric for
Figure 15. Pressure coefficient vs. $x/L$ at $\theta = 41^\circ, 101^\circ, 161^\circ, 221^\circ, 281^\circ$ and $341^\circ$ and $\phi = 171^\circ$. 
Figure 16. Local side- and normal-force coefficients vs. $x/L$ at $\alpha = 30^\circ$, $\phi = 0^\circ, 90^\circ, 180^\circ$ and $270^\circ$.

Figure 17. Local side- and normal-force coefficients vs. $x/L$ at $\alpha = 35^\circ$, $\phi = 0^\circ, 90^\circ, 180^\circ$ and $270^\circ$. 
$10^\circ < \alpha < 20^\circ$. Further increase of $\alpha$ gives rise to the initial asymmetry, and the two lateral components become non-zero. The variations of $C_{Y0}$ and $C_n$ with $\phi$ have the same pattern as those for $C_{Yd}$. The normal components $C_{N0}$ and $C_m$ are essentially invariant with roll angle.

Strain-gage-balance measurements for a $20^\circ$ cone were performed by Keener et al.\textsuperscript{7} at angles of attack from $0^\circ$ to $88^\circ$, $Re = 0.4 \times 10^6$, $M = 0.25$, and one roll angle. Their results are plotted in Figs. 18 to 24 at an arbitrarily-chosen roll angle of $\phi = 90^\circ$. The agreements with the present results are good.

![Figure 18](image1.png)

**Figure 18.** Overall force and moment coefficients vs. $\phi$ at $\alpha = 0^\circ$ compared with Keener et al.\textsuperscript{7}

![Figure 19](image2.png)

**Figure 19.** Overall force and moment coefficients vs. $\phi$ at $\alpha = 10^\circ$ compared with Keener et al.\textsuperscript{7}

Figure 25 presents the maximum and minimum values of the overall force and moment coefficients versus angle of attack. The growth of the limit $C_{N0}$ and $C_m$ with $\alpha$ are linear when $\alpha < 10^\circ$ but become nonlinear when $\alpha = 10^\circ$, which indicates that the boundary-layer separation begins at $\alpha = 10^\circ$. The limit curves for $C_{Y0}(\alpha)$ and $C_n(\alpha)$ clearly show that the asymmetry onsets at $\alpha = 20^\circ$ and the limiting values increase with angle of attack nonlinearly. Figure 25 also gives the force-measurement results of Keener et al.\textsuperscript{7} Their measured side forces and yawing moments at one roll angle lie between the present maximum and minimum curves. It confirms that the limiting values are independent of the micro surface imperfections of the models, which is shown in an accompany paper.\textsuperscript{19} The maximum local side force at a given station for two different models of the same geometry specifications under the same flow conditions practically equals one another.
Figure 20. Overall force and moment coefficients vs. $\phi$ at $\alpha = 15^\circ$ compared with Keener et al.\textsuperscript{7}

Figure 21. Overall force and moment coefficient vs. $\phi$ at $\alpha = 20^\circ$ compared with Keener et al.\textsuperscript{7}

Figure 22. Overall force and moment coefficients vs. $\phi$ at $\alpha = 25^\circ$ compared with Keener et al.\textsuperscript{7}
Figure 23. Overall force and moment coefficients vs. $\phi$ at $\alpha = 30^\circ$ compared with Keener et al.\cite{Keener2006}

(a) Overall side- and normal-force coefficient
(b) Yawing- and pitching-moment coefficient

Figure 24. Overall force and moment coefficients vs. $\phi$ at $\alpha = 35^\circ$ compared with Keener et al.\cite{Keener2006}
Figure 25. Limits of Overall force and moment coefficients vs. $\alpha$ compared with Keener et al.$^7$
VII. Conclusions

Pressure measurements at 9 stations over a 20° cone at angles of attack up to 35° over the entire range of roll angles in 5° intervals are performed in a low-turbulence, large-scale, low-speed wind tunnel with a rigid support. The results are obtained under Reynolds number of $0.3 \times 10^6$ based on the cone-base diameter, and Mach number of 0.09.

Under the above flow conditions, there are three angle of attack stages in the range of $\alpha \leq 35°$ that classify the type of side force variation with roll angle.

Stage 1: $10° < \alpha < 20°$, symmetric separation with no side force;
Stage 2: $20° < \alpha < 35°$, asymmetric separation with continuous-wave side-force variation with roll angle;
Stage 3: $\alpha = 35°$, asymmetric separation with nearly square-wave side-force variation with roll angle.

The asymmetry onset is related to the appearance of a second suction peak in the circumferential pressure distribution. The second suction peak is caused by a nearby separated vortex. It enhances the first suction peak and delays the separation on the same side of the body. The pressure asymmetry over the body corresponds to an asymmetric vortex pair in the leeward side of the body.

The experimental results show that there may exist a number of possible symmetric and asymmetric separated vortex configurations at a given angle of attack and zero sideslip. The vortex configurations result from the interaction between the separated vortex pair and the boundary conditions on the surface of the cone. Each asymmetric vortex configuration is accompanied by an exact mirror image as the body is axi-symmetric and inclined at zero sideslip.

The variation of side force with roll angle of the cone indicates that a stable vortex configuration occurs under the perturbations of micro surface imperfections of the model. The micro surface imperfections of the body trigger the occurrence of the stable vortex configuration. However, they cannot affect the vortex configuration, since all possible vortex configurations are independent of the micro surface imperfections of the model.

In Stage 2 there are an infinite number of possible vortex configurations. In Stage 3 there are nearly only two possible vortex configurations which are mirror images to each other. The continuous side-force variation with roll angle responds to the local levels of micro surface imperfections at each azimuthal angle around the model. The nearly square side-force variation is established by the dominant features of the surface imperfections. In both stages the zero crossings of the side force are located at about the same roll angles and the directions of the side force are generally the same at the same roll angles. How the micro surface imperfections of the model actually act as the instability trigger in each stage is still not quite clear.

The micro surface imperfections of a model are random and unpredictable. The variation pattern of the side force versus roll angle obtained for a model has no general significance. However, the following flow features observed in the experiments are independent of micro surface imperfections of the model, and, thus, can be applied generally to cones of the same apex angle under the same angle of attack and flow conditions.

1. Symmetric separation onsets at the angle of attack of the semi-apex angle of the cone, and asymmetric separation onsets at the angle of attack of the apex angle.
2. The asymmetry is caused by the hydrodynamic instability of the separated vortices.
3. Asymmetric pressure starts from the apex of the cone and persists down the body.
4. The surface pressures in the separated flows are not constant along rays from the apex of the cone.
5. The maximum local side force at a given station and the maximum overall side-force and yawing-moment over all roll angles are unique at a given angle of attack.
6. Every positive local side force is associated with a negative local side force with the same magnitude at different roll angles on a given station at a given angle of attack.
7. The local normal force at a given station is practically invariable with roll angle at a given angle of attack.
Appendix

The pressure coefficients $c_p$ versus meridian angle $\theta$ at angle of attack $\alpha = 0 \sim 35^\circ$, $V = 30$ m/s, Stations 1 through 9 and all roll angles in $9^\circ$ intervals, are presented in the following FIGURES and PAGES.

FIGURE 1. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 1, PAGES 1-5.

FIGURE 2. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 2, PAGES 6-10.

FIGURE 3. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 3, PAGES 11-15.

FIGURE 4. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 4, PAGES 16-20.

FIGURE 5. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 5, PAGES 21-25.

FIGURE 6. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 6, PAGES 26-30.

FIGURE 7. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 7, PAGES 31-35.

FIGURE 8. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 8, PAGES 36-40.

FIGURE 9. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 9, PAGES 41-45.

FIGURE 10. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 1, PAGES 46-50.

FIGURE 11. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 2, PAGES 51-55.

FIGURE 12. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 3, PAGES 56-60.

FIGURE 13. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 4, PAGES 61-65.

FIGURE 14. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 5, PAGES 66-70.

FIGURE 15. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 6, PAGES 71-75.

FIGURE 16. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 7, PAGES 76-80.

FIGURE 17. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 8, PAGES 81-85.

FIGURE 18. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 9, PAGES 86-90.

FIGURE 19. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 1, PAGES 91-95.

FIGURE 20. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 2, PAGES 96-100.

FIGURE 21. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 3, PAGES 101-105.

FIGURE 22. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 4, PAGES 106-110.

FIGURE 23. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 5, PAGES 111-115.

FIGURE 24. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 6, PAGES 116-120.

FIGURE 25. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 7, PAGES 121-125.

FIGURE 26. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 8, PAGES 126-130.

FIGURE 27. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 9, PAGES 131-135.

FIGURE 28. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 1, PAGES 136-140.

FIGURE 29. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 2, PAGES 141-145.

FIGURE 30. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 3, PAGES 146-150.

FIGURE 31. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 4, PAGES 151-155.

FIGURE 32. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 5, PAGES 156-160.

FIGURE 33. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 6, PAGES 161-165.

FIGURE 34. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 7, PAGES 166-170.

FIGURE 35. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 8, PAGES 171-175.

FIGURE 36. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 9, PAGES 176-180.

FIGURE 37. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 1, PAGES 181-185.

FIGURE 38. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 2, PAGES 186-190.

FIGURE 39. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 3, PAGES 191-195.

FIGURE 40. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 4, PAGES 196-200.

FIGURE 41. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 5, PAGES 201-205.

FIGURE 42. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 6, PAGES 206-210.

FIGURE 43. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 7, PAGES 211-215.

FIGURE 44. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 8, PAGES 216-220.

FIGURE 45. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 9, PAGES 221-225.

FIGURE 46. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 1, PAGES 226-230.

FIGURE 47. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 2, PAGES 231-235.

FIGURE 48. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 3, PAGES 236-240.

FIGURE 49. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 4, PAGES 241-245.

FIGURE 50. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 5, PAGES 246-250.

FIGURE 51. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 6, PAGES 251-255.
FIGURE 52. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V = 30$ m/s, Station 7, PAGES 256-260.
FIGURE 53. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V = 30$ m/s, Station 8, PAGES 261-265.
FIGURE 54. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V = 30$ m/s, Station 9, PAGES 266-270.
FIGURE 55. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 1, PAGES 271-275.
FIGURE 56. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 2, PAGES 276-280.
FIGURE 57. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 3, PAGES 281-285.
FIGURE 58. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 4, PAGES 286-290.
FIGURE 59. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 5, PAGES 291-295.
FIGURE 60. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 6, PAGES 296-300.
FIGURE 61. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 7, PAGES 301-305.
FIGURE 62. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 8, PAGES 306-310.
FIGURE 63. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 9, PAGES 311-315.

Acknowledgments

The first author acknowledges support by the Doctorate Innovation Foundation of Northwestern Polytechnical University (NPU) through Grant CX200501. The authors express their gratitude to Yongwei Gao, Zhongxiang Xi, Ye Yang, Zenghong Xi, Chunsheng Xiao, and Jiangnan Hao in NPU for their valuable technical guidance and support in the wind-tunnel tests. The authors are grateful to Professor Xueying Deng of Beijing University of Aeronautics and Astronautics for using the pressure-measurement system.

References

**FIGURE 1.** $C_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 1
FIGURE 1. (Continued)
FIGURE 1. (Concluded)
FIGURE 2. $C_p$ vs. $\theta$ at $\alpha=0^\circ$, $V=30$ m/s, Station 2
FIGURE 2. (Continued)
FIGURE 3. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 3
FIGURE 3. (Continued)
FIGURE 4. $c_p$ vs. $\theta$ at $\alpha=0^\circ$, $V=30$ m/s, Station 4
FIGURE 4. (Concluded)
FIGURE 5. $c_p$ vs. $\theta$ at $\alpha=0^\circ$, $V=30$ m/s, Station 5
FIGURE 5. (Continued)
FIGURE 5. (Continued)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{Graphs showing \( C_{p0} \) vs. \( \theta \) for different values of \( \phi \).}
\end{figure}
Figure 6. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 6
FIGURE 6. (Continued)
FIGURE 7. $c_p$ vs. $\theta$ at $\alpha = 0^\circ$, $V = 30$ m/s, Station 7
\[ \theta (\degree) \]

\[ \phi = 216^\circ \]

\[ \phi = 225^\circ \]

\[ \phi = 234^\circ \]

\[ \phi = 243^\circ \]

\[ \phi = 252^\circ \]

\[ \phi = 261^\circ \]

\[ \phi = 270^\circ \]

\[ \phi = 279^\circ \]

FIGURE 7. (Continued)
\[ \phi = 0^\circ \]

\[ \phi = 9^\circ \]

\[ \phi = 18^\circ \]

\[ \phi = 27^\circ \]

\[ \phi = 36^\circ \]

\[ \phi = 45^\circ \]

\[ \phi = 54^\circ \]

\[ \phi = 63^\circ \]

FIGURE 8. \( c_p \) vs. \( \theta \) at \( \alpha = 0^\circ \), \( V = 30 \text{ m/s} \), Station 8

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FIGURE 8. (Concluded)
FIGURE 9. \( c_p \) vs. \( \theta \) at \( \alpha = 0^\circ \), \( V = 30 \text{ m/s} \), Station 9.
FIGURE 9. (Continued)
FIGURE 9. (Continued)
FIGURE 9. (Continued)
FIGURE 10. $c_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V = 30$ m/s, Station 1
FIGURE 10. (Continued)
FIGURE 10. (Continued)
FIGURE 11. $C_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V = 30$ m/s, Station 2
FIGURE 11. (Continued)

\[ θ (°) \]

\[ c_p \]

\[ φ = 72° \]

\[ φ = 81° \]

\[ φ = 90° \]

\[ φ = 99° \]

\[ φ = 108° \]

\[ φ = 117° \]

\[ φ = 126° \]

\[ φ = 135° \]

\[ θ (°) \]

\[ \theta (°) \]

\[ θ (°) \]

\[ θ (°) \]

\[ θ (°) \]

\[ θ (°) \]

\[ θ (°) \]

\[ θ (°) \]
FIGURE 11. (Concluded)
FIGURE 12. $C_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V=30$ m/s, Station 3
FIGURE 12. (Continued)
FIGURE 12. (Continued)
FIGURE 13. $C_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V = 30$ m/s, Station 4
FIGURE 13. (Continued)
FIGURE 13. (Concluded)
FIGURE 14. $C_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V = 30$ m/s, Station 5
FIGURE 15. $c_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V=30$ m/s, Station 6
FIGURE 15. (Continued)
FIGURE 16. $C_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V=30$ m/s, Station 7
\[ \theta (\degree) \]

\[ \phi = 216 \degree \]

\[ \phi = 225 \degree \]

\[ \phi = 234 \degree \]

\[ \phi = 243 \degree \]

\[ \phi = 252 \degree \]

\[ \phi = 270 \degree \]

\[ \phi = 279 \degree \]
FIGURE 17. $C_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V = 30$ m/s, Station 8
FIGURE 17. (Continued)
FIGURE 18. $C_p$ vs. $\theta$ at $\alpha = 10^\circ$, $V=30$ m/s, Station 9
\[ \theta \] (°)

\[ c_p \]

\[ 0 \]

\[ 60 \]

\[ 120 \]

\[ 180 \]

\[ 240 \]

\[ 300 \]

\[ 360 \]

\[ -1.5 \]

\[ -1 \]

\[ -0.5 \]

\[ 0 \]

\[ 0.5 \]

\[ 1 \]

\[ \phi = 216° \]

\[ \phi = 225° \]

\[ \phi = 234° \]

\[ \phi = 243° \]

\[ \phi = 252° \]

\[ \phi = 261° \]

\[ \phi = 270° \]

\[ \phi = 279° \]

FIGURE 18. (Continued)

PAGE 89
FIGURE 18. (Concluded)
FIGURE 19. $C_p$ vs. $\Theta$ at $\alpha = 15^\circ$, $V = 30$ m/s, Station 1
FIGURE 19. (Continued)
FIGURE 19. (Continued)
FIGURE 19. (Concluded)
FIGURE 20. $C_p$ vs. $\theta$ at $\alpha = 15^\circ$, $V = 30\ m/s$, Station 2
FIGURE 20. (Continued)
FIGURE 20. (Continued)
FIGURE 20. (Concluded)
FIGURE 21. $c_p$ vs. $\theta$ at $\alpha = 15^\circ$, $V = 30$ m/s, Station 3
FIGURE 22. $c_p$ vs. $\theta$ at $\alpha = 15^\circ$, $V = 30$ m/s, Station 4

PAGE 106
FIGURE 22. (Continued)
FIGURE 22. (Continued)
FIGURE 22. (Concluded)
FIGURE 23. $C_p$ vs. $\theta$ at $\alpha = 15^\circ$, $V = 30$ m/s, Station 5
FIGURE 23. (Continued)
\[ \theta (\degree) \]

\[ c_p \]

\[ \phi = 144^\degree \]

\[ \phi = 153^\degree \]

\[ \phi = 162^\degree \]

\[ \phi = 171^\degree \]

\[ \phi = 180^\degree \]

\[ \phi = 189^\degree \]

\[ \phi = 198^\degree \]

\[ \phi = 207^\degree \]
FIGURE 23. (Concluded)
FIGURE 24. $C_p$ vs. $\theta$ at $\alpha = 15^\circ$, $V = 30$ m/s, Station 6
FIGURE 24. (Continued)
FIGURE 24. (Continued)
FIGURE 25. \( c_p \) vs. \( \theta \) at \( \alpha = 15^\circ \), \( V = 30 \text{ m/s} \), Station 7
FIGURE 25. (Continued)
FIGURE 26. $c_p$ vs. $\theta$ at $\alpha = 15^\circ$, $V = 30$ m/s, Station 8
FIGURE 26. (Continued)
FIGURE 27. $C_p$ vs. $\theta$ at $\alpha = 15^\circ$, $V=30$ m/s, Station 9
FIGURE 27. (Continued)
FIGURE 27. (Continued)
FIGURE 27. (Concluded)
FIGURE 28. $C_p$ vs. $\theta$ at $\alpha = 20^\circ$, $V = 30$ m/s, Station 1
FIGURE 28. (Continued)
FIGURE 28. (Continued)
FIGURE 29. $c_p$ vs. $\theta$ at $\alpha=20^\circ$, $V=30$ m/s, Station 2
FIGURE 29. (Continued)
FIGURE 29. (Continued)
FIGURE 29. (Concluded)
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure30}
\caption{$c_p$ vs. $\theta$ at $\alpha=20^\circ$, $V=30$ m/s, Station 3}
\end{figure}
FIGURE 31. $C_p$ vs. $\theta$ at $\alpha=20^\circ$, $V=30$ m/s, Station 4
FIGURE 32. $c_p$ vs. $\theta$ at $\alpha=20^\circ$, $V=30$ m/s, Station 5
FIGURE 32. (Continued)
FIGURE 33. $C_p$ vs. $\theta$ at $\alpha = 20^\circ$, $V = 30$ m/s, Station 6
FIGURE 33. (Continued)
FIGURE 33. (Continued)
FIGURE 33. (Concluded)
FIGURE 34. $c_p$ vs. $\theta$ at $\alpha = 20^\circ$, $V = 30$ m/s, Station 7
FIGURE 34. (Continued)
FIGURE 34. (Continued)
FIGURE 35. $c_p$ vs. $\theta$ at $\alpha = 20^\circ$, $V = 30$ m/s, Station 8
FIGURE 35. (Continued)
FIGURE 35. (Continued)
FIGURE 35. (Concluded)
FIGURE 36. $C_p$ vs. $\theta$ at $\alpha = 20^\circ$, $V = 30$ m/s, Station 9
FIGURE 36. (Continued)
FIGURE 36. (Continued)
FIGURE 36. (Concluded)
FIGURE 37. $c_p$ vs. $\theta$ at $\alpha = 25^\circ$, $V = 30$ m/s, Station 1
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure37_continued}
\caption{Continued)}
\end{figure}
FIGURE 37. (Concluded)
FIGURE 38. $C_p$ vs. $\theta$ at $\alpha = 25^\circ$, $V = 30 \text{ m/s}$, Station 2
FIGURE 38. (Continued)
FIGURE 38. (Continued)
FIGURE 38. (Concluded)
FIGURE 39. $c_p$ vs. $\theta$ at $\alpha = 25^\circ$, $V = 30$ m/s, Station 3
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig39}
\caption{(Continued)}
\end{figure}
FIGURE 39. (Continued)
FIGURE 39. (Continued)
FIGURE 40. $C_p$ vs. $\theta$ at $\alpha = 25^\circ$, $V = 30$ m/s, Station 4
FIGURE 40. (Continued)
FIGURE 40. (Concluded)
FIGURE 4.1. $C_p$ vs. $\theta$ at $\alpha = 25^\circ$, $V=30$ m/s, Station 5
FIGURE 41. (Continued)
FIGURE 42. \( c_p \) vs. \( \theta \) at \( \alpha = 25^\circ \), \( V = 30 \text{ m/s} \), Station 6
FIGURE 42. (Continued)
\[
\begin{array}{c}
\phi = 144^\circ \\
\phi = 153^\circ \\
\phi = 162^\circ \\
\phi = 171^\circ \\
\phi = 180^\circ \\
\phi = 189^\circ \\
\phi = 198^\circ \\
\phi = 207^\circ \\
\end{array}
\]
FIGURE 43. $c_p$ vs. $\theta$ at $\alpha=25^\circ$, $V=30$ m/s, Station 7
FIGURE 42. (Continued)
FIGURE 44. $c_p$ vs. $\theta$ at $\alpha = 25^\circ$, $V=30$ m/s, Station 8
\[
\hat{c}_p(\theta) \quad \phi = 72^\circ
\]

\[
\hat{c}_p(\theta) \quad \phi = 81^\circ
\]

\[
\hat{c}_p(\theta) \quad \phi = 90^\circ
\]

\[
\hat{c}_p(\theta) \quad \phi = 99^\circ
\]

\[
\hat{c}_p(\theta) \quad \phi = 108^\circ
\]

\[
\hat{c}_p(\theta) \quad \phi = 117^\circ
\]

\[
\hat{c}_p(\theta) \quad \phi = 126^\circ
\]

\[
\hat{c}_p(\theta) \quad \phi = 135^\circ
\]
FIGURE 44. (Concluded)
FIGURE 45. $c_p$ vs. $\theta$ at $\alpha=25^\circ$, $V=30$ m/s, Station 9
FIGURE 45. (Continued)
FIGURE 46. $C_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V=30$ m/s, Station 1
FIGURE 46. (Concluded)
FIGURE 47. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V=30$ m/s, Station 2
FIGURE 47. (Continued)

\( \theta (\degree) \)

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( C_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>72°</td>
<td><img src="image1" alt="Graph 1" /></td>
</tr>
<tr>
<td>90°</td>
<td><img src="image2" alt="Graph 2" /></td>
</tr>
<tr>
<td>108°</td>
<td><img src="image3" alt="Graph 3" /></td>
</tr>
<tr>
<td>117°</td>
<td><img src="image4" alt="Graph 4" /></td>
</tr>
<tr>
<td>126°</td>
<td><img src="image5" alt="Graph 5" /></td>
</tr>
<tr>
<td>135°</td>
<td><img src="image6" alt="Graph 6" /></td>
</tr>
</tbody>
</table>

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FIGURE 47. (Concluded)
FIGURE 48. $C_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V = 30$ m/s, Station 3
FIGURE 48. (Continued)
FIGURE 48. (Continued)
FIGURE 48. (Continued)
FIGURE 48. (Concluded)
FIGURE 49. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V = 30$ m/s, Station 4
FIGURE 49. (Continued)
FIGURE 49. (Continued)
FIGURE 50. $C_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V = 30$ m/s, Station 5
FIGURE 50. (Continued)
FIGURE 50. (Concluded)
\[ \theta (°) \]

\[ c_p \]

\[ \phi = 0° \]

\[ \phi = 9° \]

\[ \phi = 18° \]

\[ \phi = 27° \]

\[ \phi = 36° \]

\[ \phi = 45° \]

\[ \phi = 54° \]

\[ \phi = 63° \]

FIGURE 51. \( C_p \) vs. \( \theta \) at \( \alpha = 30° \), \( V = 30 \) m/s, Station 6

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FIGURE 51. (Continued)
FIGURE 51. (Concluded)
FIGURE 52. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V = 30$ m/s, Station 7.
FIGURE 52. (Continued)
FIGURE 53. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V=30$ m/s, Station 8
FIGURE 53. (Continued)
FIGURE 53. (Continued)
FIGURE 53. (Concluded)

\[ \phi = 288^\circ \]

\[ \phi = 297^\circ \]

\[ \phi = 306^\circ \]

\[ \phi = 315^\circ \]

\[ \phi = 324^\circ \]

\[ \phi = 333^\circ \]

\[ \phi = 342^\circ \]

\[ \phi = 351^\circ \]
FIGURE 54. $c_p$ vs. $\theta$ at $\alpha = 30^\circ$, $V=30$ m/s, Station 9
FIGURE 54. (Continued)
FIGURE 54. (Continued)
FIGURE 54. (Concluded)
FIGURE 55. $C_p$ vs. $\theta$ at $\alpha=35^\circ$, $V=30$ m/s, Station 1

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FIGURE 56. $C_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V=30$ m/s, Station 2
\[ \phi = 144^\circ \]

\[ \phi = 153^\circ \]

\[ \phi = 162^\circ \]

\[ \phi = 171^\circ \]

\[ \phi = 180^\circ \]

\[ \phi = 189^\circ \]

\[ \phi = 198^\circ \]

\[ \phi = 207^\circ \]
FIGURE 56. (Continued)
\[ c_p \] vs. \( \theta \) at \( \alpha = 35^\circ, V = 30 \text{ m/s}, \text{ Station 3} \]
FIGURE 57. (Continued)
FIGURE 57. (Continued)
FIGURE 58. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 4
FIGURE 58. (Continued)
FIGURE 58. (Continued)
FIGURE 58. (Concluded)
FIGURE 59. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V=30$ m/s, Station 5
FIGURE 59. (Continued)
FIGURE 60. $C_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 6
FIGURE 60. (Continued)
FIGURE 60. (Continued)
FIGURE 61. $C_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V=30$ m/s, Station 7
FIGURE 61. (Continued)
FIGURE 62. $C_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 30$ m/s, Station 8
FIGURE 62. (Continued)
FIGURE 62. (Continued)
FIGURE 63. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V=30$ m/s, Station 9
FIGURE 63. (Continued)