Design of Wing-Body Configurations to Delay the Onset of Vortex Asymmetry

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The stability analysis developed by Cai, Liu, and Luo (J. of Fluid Mech., vol. 480, 2003, pp.65-94) is augmented by a numerical conformal mapping technique in order to extend its applicability to problems with complex geometry. The extended method is applied to investigate the stability of symmetric vortices over wing-body combinations with various designs of the cross-sectional geometry through morphing a basic profile formed by a flat-plate wing and a circular-cone center-body. The effect of the thickness of the wing and the body, bending the wing tips, and contouring the cross-sectional profiles of the wing and the body are studied systematically. Through this process, it is discovered that a cross-sectional profile like that of a disturbed cobra has the largest stability range up to very high angles of attack for the symmetric vortices over a slender conical forebody.

I. Introduction

One most interesting phenomenon associated with high-angle-of-attack aerodynamics is the sudden onset of vortex asymmetry on the forebody of an air vehicle in symmetric flight. One of the first observations of the onset of vortex asymmetry was reported in 1951 by Allen and Perkins.1 Interest in the phenomenon has been intensified since the late 1970's as concepts for highly maneuverable aircraft have been developed. These high-performance aircraft are expected to operate routinely at angles of attack at which vortex asymmetric is known to occur. When vortex asymmetry occurs, the aerodynamic, stability, and control characteristics of the vehicle change dramatically. In the mean time, the conventional aerodynamic controls become ineffective due to the vortex wakes generated by the forebody. The subject has been reviewed by Hunt,2 Ericsson and Reding,3 and Champigny.4

High-angle-of-attack flow control is most effective when applied in the region close to the tip of the forebody. The presence of two closely-spaced vortices around the pointed forebody at high angles of attack enhances the effectiveness. Compared with that on wings, control on the forebody is required over a much small area and thus physical requirements such as size and weight should be much smaller. The lengthy forebody of a modern fighter further enhances the control effectiveness by providing a long moment arm. Excellent reviews of this activity can be found in papers by Malcolm5,6 and Williams.7

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AIAA 2006-3319
One of the devices to suppress the flow asymmetry is the horizontal nose strakes. Coe, Chambers and Letko\textsuperscript{9} showed by wind tunnel tests the alleviating effects of horizontal and symmetrical strakes placed close to the apex of a slender ogive forebody and a slender circular cone. It is observed that the effect of the strakes is to produce a well-defined point of separation at the leading edges of the strakes which results in a symmetrical flow field over a wide range of high angles of attack. Two sharp side edges on an otherwise smooth body can also serve to define the point of separation. Siclari\textsuperscript{9} used a Navier-Stokes solver to study the natural occurrence of separated flows over bi-parabolic and bi-wedge cones at free-stream Mach number of 1.8 and angle of attack of 20°. With increasing thickness ratio of the cross section of the conical bodies, the originally symmetric vortex pair separated from the sharp side edges becomes asymmetric at a thickness ratio of 0.5 and 0.6 for the bi-parabolic and bi-wedge cones, respectively.

Since the apex portion of any slender pointed body is nearly a conical body, high angle-of-attack flow about slender conical wing-body combinations were studied in a recent paper\textsuperscript{10} using the analytic method developed by Cai, Liu and Luo.\textsuperscript{11} The theoretical analysis is based on an eigenvalue analysis on the motion of the vortices under small temporal perturbations, which pertains to the absolute-type of instability. The theoretical results agree well with available experimental observations and have been corroborated by numerical computations of a three-dimensional time-accurate Euler solver.\textsuperscript{12}

The aim of the present paper is to manipulate the aerodynamic configuration for keeping the stationary symmetric vortex pair stable at high angles of attack by the analytic method. In the following sections, the theoretical method is summarized and a numerical conformal mapping technique from the literature is described. A series of configurations are analyzed for the stability of the symmetric vortex pair. A most effective configuration of wing-body combination is obtained by the theoretical method and validated by an Euler solver. Lastly, conclusions are drawn.

II. Theoretical Method

In this section, the theoretical vortex-flow model and the stability analysis method developed in Refs. 11, 13 and 10 are summarized. Semi-empirical modifications to the model are made to account for the effects of the vortex core.\textsuperscript{14} The numerical conformal mapping technique developed by James\textsuperscript{15} and Halsey\textsuperscript{16} is briefly reviewed.

A. The Vortex Velocity Expression

Consider the flow past a slender conical wing-body combination at an angle of attack \( \alpha \) and sideslip angle \( \beta \) as shown in Fig. 1. The velocity of the free-stream flow is \( U_\infty \). The planform of the wing has a half vertex angle of \( \epsilon \). In a cross-sectional plane at \( z \), the wing has a half span \( s \), and the center body has a half span \( b \). The wing-body combination is assumed to be conical and of infinite length. No effects of trailing edge or body base are considered. At high angles of attack, there is either a pair of symmetric or a pair of asymmetric vortices separated from the sharp leading edges of the wing. To study the stability of the vortex flow, the theoretical model used here is mainly that of Legendre.\textsuperscript{17} The vortex model consists of one pair of concentrated vortices separated from the sharp leading edges of the wing as shown in Fig. 1. The distributed vortex sheets that connect the leading edges and the two concentrated vortices are neglected since their strength is in general much smaller than that of the two concentrated vortices. The two concentrated vortices are assumed to be conical rays from the body apex \( O \). Secondary separation vortices, if any, are weak and thus also neglected. Vortex breakdown is not considered. The flow is assumed to be steady, inviscid, incompressible, conical, and slender. The flow is irrotational except at the center of the isolated vortices.

The inviscid incompressible flow considered in the above model is irrotational except at the lines of the isolated vortices. The governing equation for the velocity potential is the three-dimensional Laplace equation with zero normal flow velocity on smooth body surfaces, and Kutta conditions at sharp edges as boundary conditions. By the principle of superposition, the flow around the body can be obtained by solving the following two flow problems:
Flow problem 1: The flow due to the normal components of the free-stream velocity, $U_x = U_\infty \cos \beta \sin \alpha$ and $U_y = U_\infty \sin \beta$.

Flow problem 2: The flow due to the axial component of the free-stream velocity, $U_z = U_\infty \cos \beta \cos \alpha$.

Both subject to the boundary conditions. The first flow problem is solved by a conformal mapping $\zeta = \zeta(Z)$ that maps the body contour in the plane $Z = x + iy$ to a circle of unit radius in an uniform flow of velocity $(U_x, U_y)$ in the plane $\zeta = \xi + iy$. The second problem is solved by the condition of conical flow in which the flow is invariant along rays emanating from the apex.

In this paper, the theoretical flow model of Ref. 11 is modified to account for the vortex-core effects. For the vortex-core effects, a line sink of strength $Q_\epsilon$ is added to each line vortex $\Gamma$, and in the meantime the free-stream velocity component along the body axis $U_z$ in the second flow problem is augmented by a factor $(1 + K_\epsilon)$ where $K_\epsilon > 0$. $Q_\epsilon$ and $K_\epsilon$ are related to the strength of the vortex considered $\Gamma$ by a semi-empirical method. $Q_\epsilon = -q_\epsilon \Gamma$ and $K_\epsilon = \kappa (\Gamma / (2\pi s U_x))^2$, where $q_\epsilon = 0.02$ and $\kappa = 0.3$. Reference 14 gives the detail of the derivations and verifications.

Let $Z_1$ (or $\zeta_1$ on the transformed plane) be the location of the first vortex. Following Reference 13 the complex velocity at $Z_1$ is as follows.

$$u_1 - iv_1 = \left( \frac{\nabla u}{\zeta_1} - \frac{\bar{\Gamma}_1}{2\pi i} \left( \frac{1}{\zeta_1 - 1/\bar{\zeta}} \right) \right)$$

$$- \frac{\Gamma_2}{2\pi} \left( \frac{1}{\zeta_1 - \bar{\zeta}} - \frac{1}{\zeta_1 - 1/\bar{\zeta}} \right)$$

$$- \frac{q_\epsilon \Gamma_2}{2\pi} \left( \frac{1}{\zeta_1 - \bar{\zeta}} + \frac{1}{\zeta_1 - 1/\bar{\zeta}} - \frac{1}{\zeta_1} \right)$$

$$- \frac{q_\epsilon \Gamma_1}{2\pi} \left( \frac{1}{\zeta_1 - \bar{\zeta}} - \frac{1}{\zeta_1} \right) \left( \frac{d\zeta}{dZ} \right)_1$$

$$- \frac{(i - q_\epsilon) \Gamma_1}{4\pi} \left( \frac{d^2 Z}{d\zeta^2} \right)_1 \left( \frac{d\zeta}{dZ} \right)_1$$

$$- \frac{(1 + K_\epsilon) U_x \nabla u}{sK} + \frac{1}{2\pi} \sum_{j=1}^{N} \frac{Q_j}{Z_1 - Z_j}$$

(1)

where the overbar denotes complex conjugate; $U_n = U_x(1 + iK_s)$; $K = \tan \alpha / \tan \epsilon$ is the Sychev\textsuperscript{18} similarity parameter; $K_s = \tan \beta / \sin \alpha$ is the sideslip similarity parameter. The last two terms on the right-hand
side are the solution of the flow problem 2, and the other terms are the solution of the first flow problem. 

$Q_j (j = 1, 2, \ldots N)$ are the strengths of the point sources at the points $Z_j$ to be determined by $N$ simultaneous equations of the boundary conditions at $N$ points on the body contour in the augmented axial flow. The subscript 1 denotes the values at vortex point $Z_1$ (or $\zeta = \zeta_1$). A similar expression is obtained for the complex velocity at the center of the other vortex denoted by $Z_2$ (or $\zeta_2$).

It is noted that $U_\infty$ is the only parameter having the velocity dimension emerging on the right-hand side of the vortex velocity expression, and thus $U_\infty$ is chosen for the normalization purpose in this analysis.

Only vortex configurations (locations and strengths of the vortices) that result in zero flow velocities at the two vortex centers can exist in a steady flow. These locations of the vortices are called the stationary positions. The stationary positions $Z_1$ and $Z_2$, and strengths $\Gamma_1$ and $\Gamma_2$ of the two vortices in the above model are determined by solving a set of four algebraic equations by a Newton iteration method. Among the four equations, two equations set the vortex velocities to be zero, and the other two equations set the flow velocities at the separation points to be zero or finite.

In this paper, the side-slip angle $\beta$ is taken to be zero.

B. Numerical Conformal Mapping

The method of numerical conformal mapping is described here. The cross section contour considered has two sharp edges at points $Z_A$ and $Z_B$ with the same included angle $\nu$, and is smooth elsewhere. Two steps are adopted to map the body contour in the plane $Z = x + iy$ to a circle of unit radius in uniform flow of velocity $(U_x, U_y)$ in the plane $\zeta = \xi + i\eta$.

Step 1 is a corner removing mapping. The Karman-Trefftz transformation is used to map the body contour with corners in the plane $Z$ to a smooth contour in the plane $\chi$.

\[
\frac{Z - Z_A}{Z - Z_B} = \left( \frac{\chi - \mu Z_A}{\chi - \mu Z_B} \right)^{1/\mu}
\]

(2)

where $\mu = 1/(2 - \nu/\pi)$.

Step 2 is a smooth contour to unit circle mapping. The function to map the smooth contour in the plane $\chi$ to the unit circle in the plane $\zeta$ is expressed in an infinite-series form.

\[
\chi = \zeta + c_0 + \frac{c_1}{\zeta} + \frac{c_2}{\zeta^2} + \ldots
\]

(3)

where $c_0, c_1, c_2, \ldots$ are constant complex numbers to be calculated numerically.

C. Stability under Small Perturbations

When a vortex pair is slightly perturbed from their stationary positions and then released, its motion follows the vortex velocity. The increments of its coordinates as function of time are governed by a system of two linear homogeneous first-order ordinary differential equations. Define the Jacobian and divergence of the vortex velocity field $q = (u, v)$,

\[
J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}, \quad D = \nabla \cdot q = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}
\]

(4)

It is shown that the eigenvalues of this problem are

\[
\lambda_{1,2} = \frac{1}{2} \left( D_0 \pm \sqrt{D_0^2 - 4J_0} \right)
\]

(5)

where the subscript 0 denotes values at the stationary position of the considered vortex.
The eigenvalues $\lambda_1$ and $\lambda_2$ depend on the Sychev similarity parameter $K$, the sideslip similarity parameter $K_s$, and other geometric parameters, e.g. the body-wing span ratio $\gamma = b/s$. Any perturbation of the stationary positions of the vortex pair can be decomposed into a symmetric perturbation and an anti-symmetric perturbation. The maximum real part of the two eigenvalues $\lambda_1$ and $\lambda_2$ for each vortex of the stationary vortex pair under small symmetric or anti-symmetric perturbations is used to determine stability in this analysis. A positive value of this variable means perturbation growth (unstable), a negative value means perturbation decay (stable), and a zero value means perturbation remain (neutrally stable). A vortex pair is stable if and only if both vortices are stable under both symmetric and anti-symmetric perturbations.

III. Vortex Control by Configuration Manipulation

The symmetric vortex pair over a series of configurations at high angles of attack are studied by the theoretical method. The stationary symmetric vortex positions are solved and then their stability under small symmetric and anti-symmetric perturbations is investigated. These investigations finally lead to a most effective configuration of wing-body combination for suppressing flow asymmetry up to a very high angle of attack.

A. Bending the Wing Tips Upwards or downwards

It has been shown that over a slender flat-plate delta wing the symmetric vortex pair is stable at high angles of attack before vortex breakdown by experiments$^{10}$ and analyses.$^{11,20}$ How does the symmetric vortex pair behave when the cross section of the wing is bent and thickened? The thickened wings with the sharp leading edges bent upwards and downwards are studied in this subsection.

a) Bending the wing tips upwards

![Figure 2. Contour of the wing with the tips bent upwards and location of stationary symmetric vortex pair vs. $K$, $\Delta K = 0.5$.](image)

Figure 2 presents the contour of a conical thickened wing with the tips bent upwards and the location of the stationary symmetric vortex pair versus the similarity parameter $K$. The contour of the cross section is defined by two circular arcs, $x = x_c - (r_c^2 - y^2)^{1/2}$. For the upper surface, the center of the circular arc is located at $x_c/s = 1.9633$, and the radius $r_c/s = 2.2033$. For the lower surface, $x_c/s = 1.5166$, and $r_c/s = 1.8166$. The maximum thickness is 0.060s. The circle symbols denote the positions of the stationary vortices for $K = 1.0 \sim 8.0$ with an increment $\Delta K = 0.5$. The stationary symmetric vortices locate well inboard in the leeward region of the wing and move upward and further inboard with $K$.

Figure 3 presents the maximum real part of the eigenvalues for the stability of the stationary symmetric
vortex pair over the wing under small symmetric and anti-symmetric perturbations versus $K$. The symmetric vortex pair is stable when $K < 2.755$ and unstable otherwise. A similar analysis is made for a wing without bending but the same thickness (bi-circular cross section). It yields that the stable to unstable transition $K$ is beyond $K = 8.0$. In comparison with a flat-plate delta wing, non-zero thickness destabilizes the symmetric vortex pair over a slender conical wing, and upwards bending further destabilize the vortices.

b) Bending the wing tips downwards

Figure 4 shows the contour of the wing with the sharp leading edges bent downwards with the same camber as the one shown in Fig. 2. The stationary symmetric vortices over the wing are located nearly along the vertical lines passing through the wing tip points for $K = 1.0 \sim 8.0$. However, they lie inboard in the leeward region of the wing except when $K = 1.0$. When $K = 1.0$ the vortex lies very close to the upper curved surface of the wing and near the leading edge from where it is separated, and moves away upward abruptly as $K$ is increased up from 1.0.

Figure 5 presents the maximum real part of the eigenvalues for the stability of the stationary symmetric
vortex pairs over the wing with the tips bent downwards versus $K$. In contradiction with the conventional stable to unstable transition behavior, the symmetric vortex pair is unstable when $K$ is small, and stable when $K$ is large. The unstable to stable transition occurs at $K = 3.188$. Beyond $K = 3.188$, the symmetric vortex pair remains stable in the calculated region of $K$ up to $K = 8.0$. The unstable behavior when $K$ is nearly equal to 1.0 under both symmetric and anti-symmetric perturbations may be related to the separated vortex located extraordinarily close to the curved upper surface near the wing leading edge observed in Fig.4. The stable behavior when $K$ is as large as 8.0 may be related to the outboard positioning of the separated vortices. The interaction between the two separated vortices becomes weak since the distance between them is large.

We thus conclude that bending the wing tips downwards has a favorable effect on keeping the flow symmetry for large $K$, but is unfavorable when $K$ is small. To make use of the favorable effect, the destabilizing effect at low values of $K$ needs to be eliminated. This can be done by a local configuration modification as shown in Subsection C.

B. Configuring the Body with Sharp Side Edges

The body is always required to have a specified volume. The configuration considered in this subsection has two symmetric sharp side edges and the horizontal plane passing through the two edges divides the body volume into upper and lower parts. The sharp edges on the body surface are the separation locations of the vortex.

a) Larger upper volume

The contour of the body is defined by two arcs, $x = \sigma[x_c - (r_c^2 - y^2)^{1/2}]$, where $\sigma$ is a scale parameter. The lower surface is a circular arc, $\sigma = 1.0$, whose center is located on the $y$-axis at $x_c/s = 1.5166$, and whose radius $r_c/s = 1.8166$. The upper surface is an elliptic arc, $\sigma = 2.3332$, where $x_c/s = -1.5166$, and $r_c/s = 1.8166$. The x-intercept of the upper and lower surfaces are $x/s = 0.70$ and $-0.30$, respectively. Figure 6 shows the locations of stationary symmetric pairs over the body with a larger upper volume. These symmetric vortex pairs are located outboard of the leeward side of the body and moves further outboard with $K$. When $K = 1.0$ the vortex lies much closer to the sharp corner from where it is separated and moves away from the leading edge abruptly as $K$ is increased from 1.0.

Figure 7 presents the maximum real part of the eigenvalues for the stability of the stationary symmetric vortex pair versus $K$ for the above body. The symmetric vortex pair is unstable for all values of $K$. The larger upper volume destabilizes the symmetric vortex pair: The destabilization may be related to interaction
Figure 6. Contour of the body with large upper volume and sharp side edges and location of stationary symmetric vortex pair vs. $K$, $\Delta K = 0.5$.

Figure 7. Maximum real part of eigenvalues of symmetric vortex pairs over body the body with larger upper volume and sharp side edges vs. $K$, larger upper volume.
between the vortices and the upper body.

b) Larger lower volume

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure8}
\caption{Contour of body with larger lower volume and location of stationary symmetric vortex pair vs. $K$, $\Delta K = 0.5$.}
\end{figure}

If the body in Fig. 8 is turned upside-down, the lower volume becomes larger than the upper volume as shown in Fig. 8. The vortices are now located inboard in the leeward side of the body. They move upward and slightly inboard as $K$ increases. Figure 9 shows that the stationary symmetric vortex pair is stable when $K < 3.742$ and unstable otherwise in this case.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure9}
\caption{Maximum real part of eigenvalues of symmetric vortex pairs over the body with larger lower volume vs. $K$.}
\end{figure}

Consider a bi-circular arc body where $\sigma = 1.0$ for both the lower and upper bodies. A similar analysis yields that the stable to unstable transition (critical) value of $K$ is 5.865. The increase of the lower volume has a destabilizing effect. Keeping the upper surface unchanged with $\sigma = 1.0$ and changing the lower surface by varying the parameter $\sigma$ from 1.0 to 4.0 with the same $x_c/s = 1.5166$ and $r_c/s = 1.8166$, the critical value of the similarity parameter $K$ versus the scale parameter $\sigma$ are presented in Fig. 10. The critical $K$ decreases monotonically when the lower volume is increased while the upper volume remains unchanged.

From the above results, we conclude that for the same total volume a body with a larger lower volume is favored over a body with a larger lower volume for vortex stability.
The forebody of the high-performance fighter F-22 has the configuration like Fig. 8 rather than that of Fig. 6, i.e., the lower volume is larger than the upper volume. Both upper and lower surfaces are smooth, and they meet at the two sharp side edges with a larger lower volume. According to the present analysis, this configuration exhibits stable stationary symmetric vortex behavior until a rather high angle of attack.

C. Moving the Wing Tips Upwards or Downwards in a Wing-Body Combination

Consider a conical wing attached to a circular cone at the mid position, i.e., the middle plane of the wing intersects the surface of the circular cone in the horizontal plane passing through the axis of the cone. The cone-width to wing-span ratio $\gamma = 0.7$. The cross section of the wing has a maximum thickness $0.04s$ at the wing root. Keep the intersection of the wing and the circular cone surface fixed and move the sharp leading edge of the wing vertically to a height from the horizontal plane $h$ (positive when upward). To facilitate the numerical conformal mapping of the cross section contour of the wing-body combination in the plane $Z = x + iy$ to a circular contour in the plane $\zeta = \xi + i\eta$, the sharp corners formed at the conjunctions of the wing and body surface are all rounded up by a second order smooth technique so that only the two leading edges remain sharp in the entire contour of the wing-body cross section. This remark holds true for all wing-body combinations considered in this paper.

a) Curved wing with upward tips

Figure 11 depicts the contour of the wing-body combination with the wing-tips moved upwards at $h/s = 0.2$ and the location of the stationary symmetric vortex pairs for $K = 1.0 \sim 8.0$ with $\Delta K = 0.5$ and $\gamma = 0.7$. The upper and lower surfaces of the wing are circular arcs. The vortex pair lies well inboard in the leeward region of the combination body. It moves upwards nearly along vertical lines with increasing $K$. The upward movement is not uniform with respect to the increase of $K$. When $K$ increases from 2.0 to 2.5, the vortex upward movement is abrupt and covers a distance about half of the total distance over the entire range of $K$ considered. When $K$ lies in the intervals $(1.0, 2.0)$ and $(6.0, 8.0)$, the vortex upward movement is hardly seen.

Figure 12 shows the maximum real part of the eigenvalues for this configuration. The symmetric vortex pair is stable when $K < 2.1$ and unstable otherwise. The stable to unstable transition concurs with the abrupt upward movement of the vortex pair.

Similar theoretical analyses show that the stable to unstable transition value of $K$ decreases when $h/s$ increases. Therefore, the upward movement of the wing-tips destabilizes the symmetric vortex pair over the this wing-body combination. The destabilizing effect of the upward movement of the wing tips is observed.
Figure 11. Contour of the wing-body combination with the wing-tips moved upwards and locations of stationary symmetric vortex pairs vs. $K$, $\Delta K = 0.5$, $\gamma = 0.7$.

Figure 12. Maximum real part of eigenvalues of stationary symmetric vortex pairs over the wing-body combination with the wing-tips moved upwards $h/s = 0.20$ vs. $K$, $\gamma = 0.7$. 
for isolated wings in Subsection A.

b) Curved wing with downward tips

![Figure 13. Contour of the wing-body combination with the wing-tips moved downwards and locations of stationary symmetric vortex pairs vs. $K$, $\Delta K = 0.5$, $\gamma = 0.7$.](image)

Consider the same wing-body combination of Fig. 11 but with the wing-tips moved downwards at $h/s = -0.2$ as shown in Fig. 13. The vortices lie outboard of the leeward region of the combination, move upwards and further outboard with increasing $K$. The vortex movement with $K$ is smooth except when $K$ increases from 1.0 to 1.5 the vortices move up abruptly. When $K = 1.0$, the vortex lies very close to the curved upper surface and near the leading edge. According to the previous experience, a destabilizing effect near $K = 1.0$ is expected.

![Figure 14. Maximum real part of eigenvalues of stationary symmetric vortex pairs over the wing-body combination with the wing-tips moved downwards $h/s = -0.2$ vs. $K$, $\gamma = 0.7$.](image)

Fig. 14 presents the eigenvalues for this configuration. The symmetric vortex pair is stable when $1.968 < K < 5.025$ and unstable otherwise. Similar to the case of isolated downwards bent wing, the symmetric vortex pair is unstable when $K$ is low. But contrary to the case of isolated wing, the stable symmetric vortex pair goes back to unstable when $K$ is large enough due to the destabilizing effect of the center circular-cone body.$^{11}$

c) Straight wing with downward tips

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From the studies in Subsection A and this subsection, the curved wing with the tips moved downwards, no matter combined with circular cone or not, has an effect of destabilizing the symmetric vortex pair for small values of $K$. This destabilizing effect is intrinsic to the convex wing surface. When $K \approx 1.0$, the stationary symmetric vortices lie very close to the convex upper surface of the wing and near the leading edges. The instability of the symmetric vortices is essentially due to the interaction between the vortex and the nearby convex wing surface. This instability situation at low $K$ can be altered by a local configuration modification of the wing. To realize this, for the wing-body combination with $\gamma = 0.7$ the convex surfaces of the wing is replaced by straight surfaces. Draw straight lines passing through the leading edges of the wing and its root corners. The straight wing has the same wingtip downward movement $h/s = -0.2$, and keeps the same maximum thickness of 0.04s at the wing root.

![Contour of the straight-wing and body combination with wing-tips moved downwards](image)

**Figure 15.** Contour of the straight-wing and body combination with wing-tips moved downwards $h/s = -0.2$ and locations of stationary symmetric vortex pairs vs. $K$, $\Delta K = 0.5$, $\gamma = 0.7$.

Figure 15 depicts the contour of the wing-body combination and the location of the stationary symmetric vortex pair. The upper and lower surfaces of the wing are straight lines. After the local geometric alteration of the wing is applied, the positions of the stationary symmetric vortices at $K = 1.0$ are raised up significantly in comparison with Fig. 13, whereas the positions for other values of $K$ remain nearly unchanged. Figure 16 presents the eigenvalues for the above configuration. Indeed, the straight wing in this case has stable symmetric vortices at low values of $K$. The symmetric vortex pair is stable when $K < 4.762$ and unstable otherwise.

**Table 1.** Comparison of the critical similarity parameters $K$ for the wing-body combinations of curved and straight wings with tips moved downwards vs. $h/s$, $\gamma = 0.7$.

<table>
<thead>
<tr>
<th>$-h/s$</th>
<th>$K_i$</th>
<th>$K_u$</th>
<th>$K_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>3.468</td>
<td>3.458</td>
</tr>
<tr>
<td>0.05</td>
<td>0.394</td>
<td>3.739</td>
<td>3.729</td>
</tr>
<tr>
<td>0.10</td>
<td>0.795</td>
<td>4.121</td>
<td>4.036</td>
</tr>
<tr>
<td>0.15</td>
<td>1.275</td>
<td>4.565</td>
<td>4.383</td>
</tr>
<tr>
<td>0.20</td>
<td>1.968</td>
<td>5.025</td>
<td>4.762</td>
</tr>
</tbody>
</table>

For the wing-body combination with curved wing, when the wing tips are moved downwards, there exist two critical values of the similarity parameter $K$: a lower value $K_i$ and an upper value $K_u$. As $K$ increases
from 1.0 monotonically, the stationary symmetric vortex pair is unstable first, it becomes stable when $K_l$ is crossed, and it returns back to unstable when $K_u$ is crossed. However, when the curved wing is replaced by a straight wing, the original instability at low values of $K$ disappears and only the stable-to-unstable transition occurs. Thus, there exists just one critical value of $K$ for the stationary vortex pair transiting from stable to unstable for the straight-wing and body combination, which is denoted by $K_m$. Table 1 lists the critical values of $K$ for the wing-body combination with curved and straight wings and $\gamma = 0.7$. The wing tips are moved downwards from $h/s = 0.0$ to $-0.2$. As $h/s$ decreases both $K_l$ and $K_u$ increases and $K_u - K_l$ decreases slightly. When $h/s = 0$, $K_l = 0$ and $K_u = 3.468$, i.e., the symmetric vortex pair is stable when $K < 3.468$ for curved wing without wing-tip downward movement. The theoretical prediction for a flat delta wing and circular cone combination of $\gamma = 0.7$ is that the symmetric vortex pair is stable when $K \leq 3.55$ (See Fig. 23 of Ref. 14.) Thus, the wing thickness has a slight destabilizing effect on the symmetric vortex pair in the case of wing-body combination. After the local geometric modification the critical $K_m$ is slightly less than the upper critical $K_u$ of the wing-body combination with curved wing for the same $h/s$. $K_m$ increases with $-h/s$, i.e., the downwards movement of the wing-tips has an effect of stabilizing the vortex pair.

We thus see that Downward movement of the wing-tips of curved wing is favorable at high angles of attack against upward movement, but unfavorable at low angles of attack. This destabilizing effect can be eliminated while keeping the favorable stability behavior at high $K$ values by straightening the curved surface of the wing by a local geometric modification.

D. Cutting off the Upper-Half Circular Cone of Wing-Body Combination

As seen in the last subsection, the presence of the full circular cone in the wing-body combination with the wing tips moved downwards, destabilizes the symmetric vortex pair at high angles of attack. It is conjectured that the destabilizing effect may be diminished by cutting off the upper half of the circular cone. In this subsection, the upper-half circular cone is removed and the wing is extended to meet at the symmetry plane of the wing-body combination. The upper and lower surfaces of the outboard portions of the wing (i.e., outboard of the body) are straight.

a) Flat-plate wing and lower-half circular cone combination

A combination of flat-plate wing and a lower half-circular cone is studied. The wing tips are level. Figure 17 depicts the contour of the flat-plate wing and the lower half-circular cone combination of $\gamma = 0.7$ and the position of the stationary symmetric vortex pair versus the similarity parameter $K$ for $K = 1.0 \sim 8.0$
with $\Delta K = 0.5$. In comparison with flat-plate delta wing and full-circular cone combination (see Fig. 22 of Ref. 14), the vortices for the present case are more inboard and more clustered.

Figure 17. Contour of the flat-plate wing and lower-half circular cone combination and location of stationary symmetric vortex pair vs. $K$, $\Delta K = 0.5$, $\gamma = 0.7$.

Figure 18. Maximum real part of eigenvalues of symmetric vortex pairs over the flat-plate wing and lower-half circular cone combination vs. $K$, $\gamma = 0.7$.

Figure 18 presents the eigenvalues over the above configuration. The symmetric vortex pair is stable when $K < 4.637$, which is indeed greater than the critical $K = 3.55$ for the flat-plate delta wing and full-circular cone combination (see Fig. 23 of Ref. 14). It is known that the stationary symmetric vortex pair over an isolated flat-plate delta wing is stable for all values of $K$ if no vortex breakdown occurs. Cutting off the upper-half circular cone of the wing-body combination is favorable for stabilizing the flow symmetry.

b) Straight wing with root moved upwards and tips moved downwards

In this section a straight wing with the wing tips moving downwards, $h/s = -0.2$, and the wing root moved upwards, $h/s = 0.5$, is combined with the lower half-circular cone, $\gamma = 0.7$.

Figure 19 depicts the cross section of a wing-body combination with the upper-half circular cone cut off and wing tips moved downwards at $h/s = -0.2$, and wing root moved upwards at $h/s = 0.5$, and the position of the stationary symmetric vortex pair versus the similarity parameter $K$ for $K = 1.0 \sim 8.0$ with $\Delta K = 0.5$, 
\( \gamma = 0.7 \). The symmetric vortex lies nearly on the boundary of the leeward region of the combination. In comparison with the previous wing-body combination with a flat-plate wing (See Fig. 17), the vortices for the present case are more outboard.

Figure 20 presents the eigenvalues for this configuration. The symmetric vortex pair for this case is stable when \( K < 4.7 \), which is about the same as that of the case, with flat-plate wing (see Fig. 18). The introduction of the wing-tip downward movement does not bring about a raise of the critical \( K \) as desired. This is attributed to the presence of the bigger upper body in the present case.

E. Configuring Wing-Body Combination as the Disturbed-Cobra Fore-body

From the above sections, we find that the downward movement of wing tips makes the stationary symmetric vortex pair stable at large angles of attack. The flat upper and lower surfaces of the wing tips keep the vortices stable at lower angles of attack. Cutting off the upper volume of the body increases the critical value of \( K \) for the stable-to-unstable transition. Based upon the above knowledge and an intuition from the
forebody of a disturbed cobra, a cobra-like wing-body combination is configured. When disturbed, the cobra assumes a threatening position, raising the front part of its body while expanding a hood near its head. The snake creates its hood by expanding its movable neck ribs, which stretches out the loose skin around its neck.

![Figure 21. Contour of cobra-like wing-body combination and location of stationary symmetric vortex pair vs. $K$, $\Delta K = 0.5$, with $\psi = -33.69^\circ$ and $\gamma = 0.7$.](image)

Thus the contour of the cobra-like wing-body combination is created as shown in the Fig. 21. The upper contour of the wing is formed with an elliptic arc smoothly connected to a straight line at $y/s = 0.7$. The lower surface of the wing is a straight line. The body-width to wing-span ratio is $\gamma = 0.7$. The $y$-intersect of the upper surface of the wing is $x/s = 0.3$ and the $y$-intersect of lower surface of the body is $x/s = -0.7$. The straight wing tips are moved downwards with an angle of $\psi = -33.69^\circ$.

Figure 21 depicts the contour of the cobra-like wing-body combination and the position of the stationary symmetric vortex pair versus the similarity parameter $K$ for $K = 1.0 \sim 8.0$ with $\Delta K = 0.5$. The symmetric vortex lies inboard but close to the boundary of the leeward region of the combination. It moves upward and slightly inboard with $K$.

![Figure 22. Maximum real part of eigenvalues of symmetric vortex pairs over cobra-like wing-body combination vs. $K$, $\psi = -33.69^\circ$, $\gamma = 0.7$.](image)

Figure 22 presents the eigenvalues for this cobra-like wing-body combination. The stationary symmetric vortex pair is stable from $K = 1.0$ through $K = 7.5$ and almost neutrally stable for $K > 7.5$. The cobra-like
configuration yields the largest range of stable $K$ among all configuration studied in this paper. As an example, a cobra-like wing-body combination with a wing of 82° sweep angle can hold the vortex symmetry until the angle of attack is as high as 47°, if no vortex breakdown occurs on the wing.

IV. Numerical Simulation

A. The Euler Code

To verify the analytical result, a three-dimensional, time-accurate Euler solver is used to compute the separation vortex flows studied by the theoretical method. The present solver is based on a multi-block, multigrid, finite-volume method and parallel code for the steady and unsteady Euler equations. The method uses central differencing with a blend of second- and fourth-order artificial dissipation and explicit Runge-Kutta-type time marching. The resulting code preserves symmetry.

Given symmetric initial and boundary conditions, the computations using the steady-flow mode of the Euler solver yield a stationary symmetric vortex-flow solution convergent to an 11 or higher orders of magnitude decline in the maximum residual of the continuity equation. Such a stringent convergence criterion is needed for stability studies of high angle-of-attack flows as is pointed out by Siclari and Marconi. 21 To investigate the stability of the stationary symmetric/asymmetric vortex flow solutions, the steady-state solutions are used as a new initial condition and the time-accurate Euler solver is used to simulate the flow evolution under a small temporal asymmetric perturbations.

The unsteady time-accurate computations are achieved by using a 2nd-order accurate implicit scheme with dual-time stepping. In the time-accurate Euler computation, 50 real time-steps are taken in every unit increment of $t$, where $t$ is a non-dimensional time. For each real time step, the pseudo-steady-flow computation is carried out until four or higher orders of magnitude decline in the maximum residual.

The present Euler solver was originally designed for compressible flows. It is known that the numerical solution of a compressible flow solver may not converge to the physical incompressible flow as the free-stream Mach number goes to zero. In this paper to simulate low speed flow, the free-stream Mach number is set at 0.1.

B. Conical Grids

Numerical experiments with Euler code in Reference 12 show that the conical flow assumption of the theoretical model is a good approximation for slender conical bodies at low speeds and therefore a conical grid can be used in the slender conical vortex flow computations. To resolve a nearly conical vortex flow field by a conical grid, few grid lines are needed in the longitudinal direction. However, much more fine grid lines in radial and circumferential directions are required. An extraordinarily fine grid in the cross-flow plane is needed to resolve the high vorticity regions and simulate the vortex interactions and flow instability.

The conical body is infinitely long, while the computational flow field is finite. Figure 23 shows the grid on the exit plane, $z = 7.11s$, and the grid is bounded by a circle of radius $40s$, where $s$ is the local semi-span. Only every 4th line is shown in the radial and circumferential directions for clarity. The grid is symmetric with respect to the incidence plane. Every point, except the vertex point, on the lateral boundary of the conical grid has a distance about $40s$ from the body. Therefore, the lateral boundary of the conical grid is a far-field boundary.

The grid is $5 \times 256 \times 979$ along the longitudinal, radial, and circumferential directions, consisting of 8 blocks. This grid is much finer in the radial and circumferential directions. The high density of this grid is needed to resolve the high vorticity regions and to simulate the vortex interactions.

The boundary conditions on the body surfaces are that the velocity component normal to the surface is zero. Kutta condition at the sharp leading edges of the wing is satisfied automatically with the Euler code. Characteristic-based conditions are used on the upstream boundary of the grid. On the downstream boundary, all flow variables are extrapolated.
C. Temporal Asymmetric Perturbations

Small asymmetric perturbation of a short duration is applied to the stationary Euler solutions as an instability trigger in the time-accurate computation of the flow evolution. Asymmetric perturbations are applied because they are found to be most unstable mode in the theory. The small asymmetric perturbations are temporal blowing and suction on the right and left sides of the wing upper surface, respectively. The blowing/suction slots on the upper surfaces of the wing are the narrow conical regions bounded by two rays located approximately beneath the vortex cores. The two slots are located symmetric with respect to the incidence plane. The perturbations are activated in the first time period $0 < t < 1$ of the time-accurate Euler computation, where $t$ is a non-dimensional time. In the Euler computations, the blowing/suction velocity, $V_j$ is a function of $y/s$, and $t$,

$$V_j = \begin{cases} 
V_0 \sin\left(\frac{y - y_1}{y_2 - y_1} \right) \sin(\pi t) & 0 \leq t \leq 1, y_1 \leq y \leq y_2 \\
0 & \text{otherwise}
\end{cases} \quad (6)$$

The boundary conditions on the suction and blowing area on the upper surfaces of the wing are that the outward normal velocity equals to $-V_j$ and $V_j$, respectively.

D. Numerical Verification for Cobra-Like Body

The analytical methods used in this paper have been validated for the cases of flat-plate delta wing and its combinations with circular cone in Refs. 12 and 22. The cobra-like wing-body configuration obtained from the analyses is verified by the Euler computation here. The tip portion of the wing is straight with a downwards movement. The body width and wing span ratio is $\gamma = 0.7$. The analysis predicts that the stationary symmetric vortex pair is stable from $K = 1.0$ through $K = 7.5$. To verify the theoretical prediction, a typical case is chosen: $\epsilon = 8$ and $\alpha = 40^\circ$ (or $K = 5.971$).

With the free-stream flow as the initial condition, the Euler solver for steady flow is used to compute. A convergent stationary symmetric solution is obtained. Figure 24 presents the computed contours of the longitudinal velocity component in a cross plane. In the vortex center the pressure coefficient $c_p = -19.95$ and the longitudinal velocity component $u/U_{\infty} = 4.0926$. It depicts that the vortex sheets are separated from the leading edges and rolled up into a pair of vortex core. At the vortex-core center the longitudinal velocity component reaches its maximum value about four times that of the free stream velocity. The analytical solutions of the symmetric vortex position for $K = 1.0 \sim 8.0$ are reproduced from Figure 21 for a
comparison with the computed case. The deviation of the analytic result at $K = 6.0$ (pointed at by a larger arrow) from the computed one at $K = 5.971$ is attributed to the neglect of the vortex sheets in the theory.

![Figure 24. Contour of the longitudinal velocity component on a cross flow plane and comparison of the stationary symmetric vortex center position with the theoretical results, over a cobra-like wing-body combination with $\psi = 33.690^\circ$, $\gamma = 0.7$, $\epsilon = 8^\circ$, $\alpha = 40^\circ$ (or $K = 5.971$).][1]

![Figure 25. Vortex center position ($x/s, y/s$) vs. time $t$ after a temporal disturbance is applied to the stationary symmetric vortex solution over a cobra-like wing-body combination, $\psi = 33.690^\circ$, $\gamma = 0.7$, $\epsilon = 8^\circ$, $\alpha = 40^\circ$, (or $K = 5.971$).][2]

To investigate the stability of the stationary symmetric vortex pair, the converged symmetric solution is then used as a new initial condition, and the time-accurate Euler solver is used to simulate the evolution of the original symmetric flow after a small temporal asymmetric perturbation of Eq. 6 where $y_1 = 0.75s$, $y_2 = 0.80s$, and $V_0 = 2.5U_{\infty}$, is activated in the first time period $0 < t < 1$. The two smaller arrows of opposite directions on the upper surface of the wing in Fig. 24 denote the temporal blowing and suction disturbances. The evolution of the vortex core positions ($x/s, y/s$) against the non-dimensional time $t$ for $0 \leq t \leq 25$ are recorded in Fig. 25. The initially disturbed vortex pair returns to the original symmetric position when $t > 20$. The computed result confirms the theoretical prediction that the vortices are stable.

[1]: https://example.com/image1.png
[2]: https://example.com/image2.png
V. Conclusions

The stability theory of Refs. 11-14 is further developed with the help of a numerical conformal mapping technique to study the stability of the stationary symmetric vortex pair over slender conical bodies of almost-arbitrary cross-sections at high angles of attack and low speeds. A number of typical cross-sectional configurations of slender conical wing-body combinations have been investigated systematically by the theory. The following conclusions can be drawn.

1. Downward movement of the wing tips can keep the symmetric vortex pair stable up to a higher angle of attack than upward movement. However, the tip portion of the wing has to be straight in order to avoid a possible occurrence of instability at low angles of attack.

2. Making the upper volume smaller for a body or wing-body combination can keep the symmetric vortex pair stable up to a higher angle of attack.

3. Shaping the wing-body combination as the forebody of a disturbed cobra can keep the symmetric vortex pair stable in a large range of the Sychev similarity parameter from 1.0 to 7.5.

The stable stationary symmetric vortex pair over a cobra-like body whose planform has a semi-vertex angle of 8.0° at an angle of attack of 40° is validated by the Euler computation.

References

