Supercritical Flow over Unstalled Plunging Airfoil Computed by Euler Method

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The air flow around a flapping airfoil can reach supercritical even when the freestream Mach number is as low as 0.2. The supercritical flow about a sinusoidally plunging airfoil NACA 0012 in a uniform stream parallel to the airfoil chord is studied by the inviscid version of a three-dimensional unsteady compressible Euler/Navier-Stokes flow solver. Fully attached flow is assumed. The Euler solver is validated by a Navier-Stokes solutions at freestream Mach number of 0.3 in the literature, and a dynamic-stall boundary is identified as the minimum pressure coefficient over the airfoil equal to a constant. The instantaneous load distribution is decomposed into harmonic components and compared with the classical linear oscillating-airfoil theory. The effects of the local supersonic-flow region are investigated.

I. Introduction

It has been known for many years that a flapping wing generates a thrust force. Garrick\(^1\) used Theodorsen's incompressible oscillatory flat-plate theory\(^2\) to show that a pure sinusoidally plunging airfoil produces a propelling force in the entire range of reduced frequency, the efficiency being 50 percent for infinitely rapid oscillations and 100 percent for infinitely slow flapping. At present, it is recognized that flapping wing propulsion may be more efficient than conventional propellers if applied to micro-air vehicles, because of the very low Reynolds number encountered on such vehicles. For this reason, many projects are devoted to investigate the aerodynamic characteristics of flapping airfoils.

McCroskey et al.\(^3\) observed experimentally that compressibility can play a role in pitching airfoil performance: when the angle of attack of an airfoil increases, the acceleration of the flow that occurs around the leading edge can result in flow velocities four to five times greater than the free stream value. Thus, a freestream flow of Mach number as low as 0.18 can induce sonic velocity on an airfoil pitching dynamically to high angles of attack. This compressibility effect occurs near the leading edge of the flapping airfoil.

Fung and Carr\(^4\) studied the observed behavior of the flow around a pitching-oscillating airfoil NACA 0012 with an amplitude of 10° and mean angle of attack of 15° before separation from Ref.3. They found that the parametric dependency of separation on frequency for supercritical flows is different from that for subcritical

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flows. For subcritical flows, increasing the reduced frequency delays separation of the boundary layer and, hence, allows the airfoil to attain high minimum pressure coefficient at higher angles of attack. However, for supercritical flows, the formation of a local supersonic region and the associated shock can occur at a location close to the leading edge. The vortical content of the flow is intensified due to the relatively short extension of the local supersonic region. The local outer flow and the boundary layer are no longer stable. Hence, compressibility effects pose a limit on pressure suction enhancement by increasing unsteadiness.

Tuncer, Wark and Platzer\textsuperscript{5} using a thin-layer Navier-Stokes solver calculated the mean thrust coefficient versus nondimensional plunging amplitude at various reduced frequencies. A dynamic-stall boundary where the thrust starts to drop with increasing plunging amplitude is located by approaching the boundary from the opposite sides. The computed conditions are at the free-stream Mach number of 0.3 and the Reynolds number of 10\textsuperscript{6} for a fully turbulent flow.

Very recently, Young and Lai\textsuperscript{6} found the compressibility effect occurring at high frequencies using a compressible Navier-Stokes computations. They showed that the freestream Mach number used in the simulations has an important effect on the forces predicted for both pitching and plunging oscillations, although no significant influence on the wake structures was observed in the simulations. The computed mean thrust coefficient for a plunging airfoil at freestream Mach number 0.2 deviates from the classical linear theory when the reduced frequency is high enough for a constant nondimensional maximum plunging velocity.

The objective of this study is to explore the details of the compressibility effect occurs near the leading edge of a plunging airfoil by Euler method. The Euler flow solver is described and validated against the known Navier-Stokes computational results for plunging airfoil. A dynamic stall boundary is identified. The supercritical-flow field over the plunging airfoil with no dynamic stall is examined and the dominate harmonic component of the computed load is compared with the linear theory. Lastly, conclusions are drawn.

II. Euler Solver

It is known that the Euler solver can capture automatically the inviscid wake. Although the boundary layer together with the viscous wake are absent in the Euler solutions the primary trailing-edge vortex configurations and their interactions with the moving body surface are reproduced as long as there are no significant boundary-layer separations.

The present Euler solver is based on a multi-block, multigrid, finite-volume method and parallel code for the three-dimensional, compressible steady and unsteady Euler and Navier-Stokes equations. The method uses central difference with a blend of second- and fourth-order artificial dissipation and explicit Runge-Kutta-type time marching. The coefficients of the artificial dissipation depend on the local pressure gradient. The order of magnitude of the added artificial dissipation terms is of the order of the truncation error of the basic scheme, so that the added terms have little effect on the solution in smooth parts of the flow. Near the steep gradients the artificial dissipation is activated to mimic the physical dissipation effects. Unsteady time-accurate computations are achieved by using a second-order accurate implicit scheme with dual-time stepping. The solution for each real-time step is solved by an explicit time-marching scheme in a fictitious time in which the local time stepping, residual smoothing, and multigrid techniques can be used to accelerate convergence to a steady state. The resulting code preserves symmetry.\textsuperscript{7}

On the airfoil surface the instantaneous flow normal velocity component is set equal to the local surface normal velocity component prescribed by the oscillatory motion. The far field boundary is located at 20c away from the airfoil. At the far-field inflow and outflow boundaries the flow variables are evaluated using the zero-th order Riemann invariant extrapolation. The initial condition is the airfoil starting from a position in its oscillatory motion in a free-stream flow and unique solution is obtained for any position used for the initial condition. The solver has been validated for a number of steady and unsteady cases.\textsuperscript{8-13}

The C-type grid, 1153 × 65 moving with the plunging airfoil is used for all the computations in this paper. There are 769 grid points around the airfoil, and 192 vertical grid lines and 129 horizontal grid lines in the region downstream of the airfoil. A close-up view of the grid for airfoil NACA 0012 and near wake
Figure 1. Close-up view of the grid for airfoil NACA 0012 and near wake, only every other line is plotted in the figure for clarity.
is shown in Fig.1. Only every other line is plotted in the figure for clarity. This grid yields aerodynamic forces which are grid-independent as shown by the numerical experiments in Reference 12. The numerical experiments of time-step refinement were also performed and the solutions with a step number per cycle, nstep = 64 are time-step independent. For freestream Mach number between 0.05 and 0.3, about five cycles of computations are needed for the transients to disappear.

It is known that flow bifurcation appears in pure plunge at zero mean angle of attack when the maximum instantaneous angle of attack is extremely large. Jones et al.\textsuperscript{14} reported a deflected vortex street and a non-zero time-averaged lift for the case $k = 6.15, h = 0.12$, i.e., having a maximum effective angle of attack equal to 55.9°, in water-tunnel experiments and reproduced this phenomena by a potential-flow code using different initial conditions. Lewin et al.\textsuperscript{15} also demonstrated the flow switching by a viscous-flow code. For the cases without dynamic stall, the numerical solutions of the Euler solver are independent of the initial conditions.

III. Computational Model

The airfoil NACA 0012 with chord length $c$ performs a sinusoidal plunging motion $y(t)$ perpendicular to its chord in a uniform flow of speeds $U_\infty$ parallel to its chord. The plunging movement of the airfoil is described below.

$$y(t) = y_0 \sin(\omega t).$$ \hfill (1)

where $t$ is physical time, $\omega$ and $y_0$ are the angular frequency and the amplitude of plunging motion, respectively.

Non-dimensional plunge amplitude, $h = y_0/c$. Reduced frequency, $k = \omega c/2U_\infty$. The instantaneous non-dimensional plunging velocity is

$$\dot{y}/U_\infty = 2hk \cos(\omega t).$$ \hfill (2)

where a dot denotes a differentiation with respect to $t$. The non-dimensional maximum plunging velocity, $\omega y_0/U_\infty = 2hk$. The instantaneous effective angle of attack due to pure plunging is

$$\alpha = \tan^{-1}(-\dot{y}/U_\infty).$$ \hfill (3)

The maximum effective angle of attack, $\alpha_{\text{max}} = \tan^{-1}(2hk)$ occurring at $y = 0$. The time-averaged angle of attack over a plunging cycle is zero.

The instantaneous pressure coefficient is $c_p$. The instantaneous loading coefficient is $\Delta c_p = (c_p)_{\text{lower}} - (c_p)_{\text{upper}}$. The time-averaged thrust coefficients and input power coefficient over an oscillation period are $C_T$ and $C_p$, respectively. The propulsive efficiency $\eta = C_T/C_P$.

Under the assumption of no dynamic stall, the Euler method can predict the unsteady pressure distribution and thereby provide the instantaneous and cycle-averaged lift, pitching moment, power input, and inviscid-drag over the plunging airfoil, but not the viscous profile drag. The viscous profile drag can be modeled using an interactive inviscid flow and boundary layer calculation, or approximately by quasi-steady drag polar correlations of the airfoil. In the present paper the quasi-steady skin friction of two sides of a smooth flat plate\textsuperscript{16} $C_F$ is subtracted from the inviscid thrust coefficient $C_T$ computed from the Euler method as an approximate modeling of the viscous effects. For turbulent flow, $C_F = 2 \times 0.072/Re^{1/5}$.

IV. Validation of Euler Method

The Euler method is validated by the Navier-Stokes computational results given by Tuncer et al.\textsuperscript{5} They computed the time-averaged thrust coefficient, $C_T$ versus non-dimensional plunging amplitude, $h$ for various reduced frequencies $k = 0.125 \sim 1.5$ at $M_\infty = 0.3$, $Re = 10^8$ for turbulent flow. The corresponding Euler
solutions for nonseparated cases are computed with the grid $1153 \times 65$ and $n_{step} = 64$. A flat-plate skin friction is subtracted from the inviscid thrust from the Euler solutions. Fig. 2 presents the friction-corrected Euler solutions in comparison with the Navier-Stokes solutions. Furthermore, the friction-corrected mean

![Friction-corrected thrust coefficient $C_T$ versus $h$ at various $k$ for plunging airfoil NACA 0012 computed by Euler method and linear theory and compared with Navier-Stokes solutions. $M_\infty = 0.3$, $Re = 10^6$, turbulent flow.](image1)

Figure 2. Friction-corrected thrust coefficient $C_T$ versus $h$ at various $k$ for plunging airfoil NACA 0012 computed by Euler method and linear theory and compared with Navier-Stokes solutions, $M_\infty = 0.3$, $Re = 10^6$, turbulent flow.

thrust coefficient $C_T$ and propulsive efficiency $\eta$ versus $k$ for a series of nonseparated turbulent flow cases at $M_\infty = 0.3$ and $Re = 10^6$ are computed by the Euler solver and compared with the Navier-Stokes solutions of Ref. 5 in Fig. 3. The good agreement indicates that the Euler solver are valid for attached flows in a the range of reduced frequency and plunging amplitude considered.

![Friction-corrected thrust coefficient $C_T$ and propulsive efficiency $\eta$ versus $k$ for the nonseparated flows over plunging airfoil NACA 0012 computed by Euler method and linear theory and compared with Navier-Stokes solutions, $M_\infty = 0.3$, $Re = 10^6$, turbulent flow.](image2)

Figure 3. Friction-corrected thrust coefficient $C_T$ and propulsive efficiency $\eta$ versus $k$ for the nonseparated flows over plunging airfoil NACA 0012 computed by Euler method and linear theory and compared with Navier-Stokes solutions, $M_\infty = 0.3$, $Re = 10^6$, turbulent flow.

V. Dynamic Stall Boundary

The dynamic-stall boundary for the plunging airfoil NACA 0012 at $M_\infty = 0.3$ and $Re = 10^6$ in turbulent flow in the range $k = 0.125 \sim 1.5$ was identified by Tuncer et al.\textsuperscript{5} using a Navier-Stokes code with the
Baldwin-Lomax turbulence model as the nondimensional maximum plunge velocity, \(2hk = 0.35\) which is equivalent to the maximum effective angle of attack, \(\alpha_{\text{max}} = 19.3^\circ\). In fact, it is actually not a good criterion for dynamic stall at the freestream Mach number of 0.3.

From Fig. 2 which plotted the curve \(C_T\) versus \(h\) at constant \(k\), the dynamic-stall onset boundary can be defined as the point where the slope of the curve starts to decrease with the increase of \(h\), which indicates that a significant boundary-layer separation occurs. With this definition of dynamic stall, a series of nonseparated and separated solutions approaching the dynamic-stall-onset boundary from opposite sides of the boundary are taken from Ref.5. Fig. 4 (left) presents the nondimensional maximum plunging velocity \(\omega y_0/U_\infty\) or maximum effective angle of attack \(\alpha_{\text{max}}\) versus the reduced frequency \(k\) for the separated and nonseparated cases close to the dynamic-stall boundary. The dynamic stall actually occurs at \(2hk = 0.30\) or \(\alpha_{\text{max}} = 16.7^\circ\) when \(k = 0.125\), and at \(2hk = 0.45\) or \(\alpha_{\text{max}} = 24.2^\circ\) when \(k = 1.5\). Therefore, the condition \(2hk = 0.35\) is not valid at both ends of the range of the reduced frequency, \(k = 0.125 \sim 1.5\) for \(M_\infty = 0.3\).

![Figure 4. Nondimensional maximum plunging velocity \(\omega y_0/U_\infty\) (left) and minimum pressure coefficient \((c_p)_{\text{min}}\) computed by Euler method versus reduced frequency \(k\) for plunging airfoil NACA 0012 in separated and nonseparated cases, \(M_\infty = 0.3, Re = 10^6\), turbulent flow.](image)

Since the nondimensional maximum plunging velocity \(\omega y_0/U_\infty\), or the maximum effective angle of attack \(\alpha_{\text{max}}\) along the dynamic stall boundary is actually not constant in the range of \(k\), \(2hk = 0.35\) is incapable to identify the dynamic-stall boundary. To identify the dynamic-stall boundary, the minimum pressure coefficient, \((c_p)_{\text{min}}\) over the airfoil surface on the dynamic stall boundary is considered instead of \(2hk\). The values of \((c_p)_{\text{min}}\) for the various separated and nonseparated cases close to the dynamic-stall-onset boundary from opposite sides which were obtained by the Navier-Stokes method\(^5\) are computed by the Euler solver under the assumption of no dynamic stall. Fig. 4 (right) presents the computed minimum pressure coefficient \((c_p)_{\text{min}}\) versus the reduced frequency \(k\) for the separated and nonseparated cases close to the dynamic-stall boundary by the Euler solver. It is noted that the \((c_p)_{\text{min}}\) values computed by the Euler solver for the separated cases are not the values of the real flows. They are used here to locate the dynamic-stall boundary. For the computed range \(k = 0.125 \sim 1.5\), on the dynamic-stall boundary the values of \((c_p)_{\text{min}}\) stay at roughly the same value, \(-12.5\) regardless of reduced frequency from Fig. 4 (right). Therefore, \((c_p)_{\text{min}} = -12.5\) is a better dynamic-stall boundary than \(2hk = 0.35\) for the cases considered.

For pitching-oscillating airfoil NACA 0012 at \(M_\infty = 0.3\), the dynamic-stall boundary has a similar character as the plunging-oscillating airfoil.\(^4\) At \(M_\infty = 0.3\) and \(Re = 4.2 \times 10^6\), the \((c_p)_{\text{min}}\) on the dynamic-stall boundary stays at roughly the same value, \(-9.0\), regardless of pitching frequency, amplitude, or mean angle of attack. While at \(M_\infty = 0.073\), \((c_p)_{\text{min}}\) on the dynamic-stall boundary is no more a constant and increases from \(-13\) to \(-16\) with \(k\) from 0.10 to 0.25. (See Table 1 and Fig.2 of Ref.4.) The values of \((c_p)_{\text{min}}\) at dynamic-stall onset are different for different types of motion. For pitching oscillation, there exist
angular acceleration and dynamic cambering due to angular velocity, while for plunging oscillation there exists vertical acceleration due to vertical velocity.

At $M_{\infty} = 0.3$, the compressibility plays an decisive role in the onset of dynamic stall. This will be elucidated further in the next section.

### VI. Pressure Distribution and Supercritical Flow

Under the assumption of no dynamic stall, pressure distribution over the plunging airfoil can be calculated by the Euler solver. Two nonseparated cases are considered: (1) $k = 0.125, h = 1.20$, and (2) $k = 1.5, h = 0.15$. Fig.5 presents the instantaneous pressure coefficient $c_p$ over the upper surface (left) and lower surface of the plunging airfoil NACA 0012 in an entire downstroke for the case (1) (upper) and case (2) at $M_{\infty} = 0.3$. In the left two plots, $(x/c)^{1/2}$ is used to stretch the forward portion of the airfoil chord for clarity. There

![Figure 5. Instantaneous pressure coefficient $c_p$ over upper surface versus $(x/c)^{1/2}$ (left) and lower surface versus $x/c$ of plunging airfoil NACA 0012 in downstroke computed by Euler method, $M_{\infty} = 0.3$, case (1): $k = 0.125, h = 1.20$ (upper) and case (2): $k = 1.5, h = 0.15$.](image)

exist two types of pressure distribution over plunging airfoil: leading-edge loading and uniform loading. It is the leading-edge loading pressure distribution resulting in a significant thrust. The leading-edge loading is generated by the bound vorticity over the airfoil in accompany with the induced inviscid wake vortices when the angle of attack is large and the vertical acceleration is low which happens near the neutral position.
of the plunging airfoil, $y = 0$. The uniform loading has its origin in the apparent mass with large vertical acceleration of the plunging airfoil which happens near the top and bottom positions, $y = \pm y_0$. The leading-edge-loading pressure distribution while making significant contribution to thrust when the flow is attached, is prone to induce leading-edge separation and thus causes thrust breakdown. The uniform-loading pressure distribution contributes little thrust but has an effect to relieve the leading-edge loading. Thus, the maximum effective angle of attack on the dynamic-stall boundary increases with reduced frequency as observed in the previous section. The appearance of anomalous pressure rise and down at the trailing edge are due to the non-convergence of the numerical solution of a compressible flow solver to the physical incompressible flow as the freestream Mach number goes to zero as shown in Ref. 12.

In a fraction of downstroke, a local supercritical flow region emerges near the leading edge of the upper surface in the downstroke. For case (1): $k = 0.125, h = 1.20$, the flow is supercritical when $\omega t$ lies between $7\pi/8$ and $21\pi/16$, and $(c_p)_{min} = -12.5$ occurs at $\omega t = 9\pi/8$. For case (2): $k = 1.5, h = 0.15$, supercritical flow appears for $\omega t$ between $29\pi/32$ and $43\pi/32$ and $(c_p)_{min} = -12.2$ at $\omega t = 19\pi/16$. The supercritical flow spans nearly a half of the entire downstroke in both cases. Since the flow over the plunging airfoil in up and down strokes is anti-symmetric, in total the supercritical flow sustains about half of the whole cycle.

The appearance of supercritical flow at a freestream Mach number as low as 0.3 is partly due to the continuation of attached flow to high angles of attack that are reached before separation occurs. For the cases (1) and (2), the maximum effective angles of attack are $16.7^\circ$ and $24.2^\circ$, respectively, as shown in Fig. 4 (left). They are significantly greater than the static stall angle. The static stall angle of attack is $13.7^\circ$ at $M_\infty = 0.3$ and $Re = 4.2 \times 10^6$ which was measured by McCroskey et al.3

The pressure distribution at the instant of yielding maximum suction is examined for the above two cases. Fig. 6 presents the pressure coefficient $c_p$ versus $x/c$ (left) and $(x/c)^{1/2}$ over the plunging airfoil NACA 0012 at (1) $k = 0.125, h = 1.20$, $\omega t = 9\pi/8$ and (2) $k = 1.5, h = 0.15$, $\omega t = 19\pi/16$, at $M_\infty = 0.3$. In the right figure $(x/c)^{1/2}$ is used to stretch the forward region of the pressure distributions for clarity. The critical pressure coefficient $c_p^*$ marked in Fig. 6 is calculated using the isentropic flow formula. The spatial extent of the supersonic zone is about 1.7% of the chord. However, the supersonic zone persists for half of every half cycle which corresponds to a time interval of the order of that represents chord length of travel.

The local Mach number and entropy distribution versus $(x/c)^{1/2}$ over the airfoil surfaces for the two cases considered are shown in Fig. 7 where the entropy is represented by $(p/\rho^\gamma)/(p/\rho^\gamma)_\infty$. In calculating the local Mach number and entropy, the local flow parameters are used and no isentropic formula used. An abrupt increase of the entropy appears where the local Mach number changes sharply from 1.5 to 0.6 and a shock wave does exist for both cases. Mach number contours around the leading edge of the plunging airfoil are

![Figure 6. Supercritical flow pressure distributions over plunging airfoil NACA 0012 versus $x/c$ (left) and $(x/c)^{1/2}$ computed by Euler method, $k = 0.125, h = 1.20$, $\omega t = 9\pi/8$ and $k = 1.5, h = 0.15$, $\omega t = 19\pi/16$, $M_\infty = 0.3$.](image-url)
Figure 7. Mach number and entropy distributions over plunging airfoil NACA 0012 versus $(s/c)^{1/2}$ computed by Euler method, $k = 0.125$, $h = 1.20$, $\omega t = 9\pi/8$ (left) and $k = 1.5$, $h = 0.15$, $\omega t = 19\pi/16$, $M_\infty = 0.3$.

plotted in Fig.8 for the two cases considered. The shock wave is a normal shock. The shock strength is

Figure 8. Mach number contours around the leading edge of plunging airfoil NACA 0012 computed by Euler method, $k = 0.125$, $h = 1.20$, $\omega t = 9\pi/8$ (left) and $k = 1.5$, $h = 0.15$, $\omega t = 19\pi/16$, $M_\infty = 0.3$.

strongest at the airfoil surface and decreases to zero as it penetrates into the flow field. The height extent of the normal shock is about 1% of the airfoil chord. It is interesting to note that for pitching airfoils McCroskey et al.\(^3\) found a transonic flow develops near the leading edge for $M_\infty \geq 0.18$ but did not observe any shock patterns until $M_\infty = 0.3$ in shadowgraph studies. The further growth in the maximum local Mach number above sonic conditions was found to accompany a strong tendency toward leading-edge stall, whatever the dynamic-stall behaves at the lower freestream Mach numbers. The leading-edge separation may start at the foot of the local normal shock wave, and, thus, $(c_p)_{min} = constant$ becomes an indicator of the dynamic-stall onset for the supercritical flows as shown in the previous section. In subcritical-flow region, dynamic-stall separation may start from the trailing edge and $(c_p)_{min}$ is no more a constant on the dynamic-stall boundary.
VII. Load Distribution and Comparison with Linear Theory

To compare with the linear theory, the load distribution computed by the Euler solver is decomposed into harmonic components with respect to a reference motion of the airfoil which is chosen as the downward plunging velocity. This motion parameter is related to the effective angle of attack $\alpha$ by Eq. 3 and is expressed in a form of magnitude and phase.

$$-\dot{y}/U_\infty = 2hk \cos(\omega t + \pi)$$

(4)

The aerodynamic load $\Delta c_p$ is decomposed as

$$\Delta c_p = \sum_{i=1}^{n} \Delta c_{pi} \cos(i\omega t + \phi_{pi})$$

(5)

where $\Delta c_{pi}$ and $\phi_{pi}$ are the magnitude and phase angle of the $i-th$ harmonic component, respectively.

![Graphs showing load distribution](image)

Figure 9. Magnitude of harmonic components of instantaneous load coefficient $\Delta c_{pi}$ and phase of first harmonic component $\phi_{pi}$ over plunging airfoil NACA 0012 versus $(x/c)^{1/2}$ with respect to $-\dot{y}/U_\infty$, computed by Euler solver and compared with linear theory, $k = 0.125, h = 1.20, \omega t = 9\pi/8$ (top) and $k = 1.5, h = 0.15, \omega t = 19\pi/16, M_\infty = 0.3$.

Fig. 9 presents the magnitude and phase angle of the harmonic components of $\Delta c_p$ versus $(x/c)^{1/2}$ for $k = 0.125, h = 1.20$ (top) and $k = 1.5, h = 0.15$ computed by the Euler solver and compared with the linear
theory. The magnitude of even-numbered harmonic components is negligible in comparison with that of the odd-numbered harmonic components and are not shown in the figure. The magnitude of higher odd-numbered harmonic component is not negligible. This is caused by the small local supercritical-flow region near the leading edge of the airfoil. However, they are of one order less than the first harmonic component. Thus, the first harmonic component dominates, and a comparison between the linear theory and the Euler computation based on its first harmonic component is valid. The phase of the first harmonic component is displayed in the right figures. If the flow is subcritical, all higher harmonic components of the Euler solution are negligible in comparison with its first component as shown in Ref. 17.

At \( k = 0.125 \), the first harmonic component of the load distribution \( \Delta c_p \) over the airfoil chord agrees reasonably well with linear theory in both magnitude and phase. Due to the appearance of supersonic region, the distribution of the computed load magnitude near the leading edge is quite different from the linear theory. At \( k = 0.125 \) the load distribution is nearly in phase with the reference motion. This means that the load is essentially of circulation origin. The computed aft load appears to lead that given by the linear theory.

At \( k = 1.5 \) there are important deviations of the Euler solution from the linear theory in the trailing edge region. The Euler solutions gives a nonzero aft-loading magnitude and an abrupt aft-loading phase lagging. In the linear theory, the load magnitude becomes zero when approaching the trailing edge, and the load phase angle \( \phi_p \) tends to \( 3\pi/2 \), i.e., the aft load leads the motion approximately by a right angle due to the plunging acceleration origin. In the Euler solutions the load magnitude is finite at the trailing edge and the aft load lags that of the linear theory. This deviation at the higher reduced frequency \( k = 1.5 \) from the linear theory also occurs for subcritical flows and was studied in Ref. 17. The Euler method gives qualitatively the real behavior of the unsteady trailing-edge flow at high reduced frequencies presented in the literature, for example, Refs. 6, 18 and 19. A systematic quantitative assessment of the Euler predictions on the aft-loading events by experimental measurements and Navier-Stokes computations is awaited.

VIII. Conclusions

The time-averaged thrust coefficient with a simple skin-friction correction and propulsive efficiency for a plunging airfoil NACA 0012 with no dynamic stall computed by the Euler method at freestream Mach number of 0.3 agree well with Navier-Stokes computational results in the literature. Although the thin boundary layer, its possible minor separation near the trailing edge and wake are absent in the Euler solutions, the primary separation shear layer and its rolling-up into concentrated vortices are reproduced. The Euler methods allow the computation of pressure distribution and the hysteresis effects of unsteady flow and thereby provide the instantaneous inviscid aerodynamic forces and their lag or lead with respect to the airfoil motion.

For plunging airfoil NACA 0012 at a freestream Mach number of 0.3, the present paper demonstrates that the maximum effective angle of attack on the dynamic-stall boundary actually increases with increasing reduced frequency, whereas the minimum pressure coefficients remains nearly a constant, \(-12.5\) at the onset of dynamic stall, regardless of reduced frequency and amplitude, based on the known Navier-Stokes solutions.

Similar to the case of pitching oscillation, dynamic stall of the plunging airfoil for supercritical flows is being caused by the formation of a local supersonic region and the associated shock located close to the leading edge. Hence, the minimum pressure coefficient occurred at the foot of the local shock becomes the parameter indicating the onset of dynamic-stall for supercritical flow.

Due to the appearance of the local supercritical region, the odd-numbered higher harmonic components of the load distribution close to the leading edge are not negligible relative to the first harmonic component. A finite trailing-edge pressure loading accompanying with an abrupt aft-load lagging the fore-load over the plunging airfoil at high reduced frequency is observed for the supercritical flow as for subcritical flows.
References