Control of Vortices over Slender Conical Bodies
— A Theoretical and Computational Study

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Pneumatic controls of vortices over slender conical bodies at high angles of attack and low speeds are studied by a theoretical method developed by Cai, Liu, and Luo (J. of Fluid Mech., vol. 480, 2003, pp.65-94) and verified by Euler computations. The theoretical method is based on an eigenvalue analysis on the motion of the vortices under small perturbations, which pertains to the absolute-type of instability. Steady blowing and suction are simulated by sources and sinks. A modification of the original model to account for the effects of the vortex core is implemented. The theory predicts the positions of stationary conical vortices and their stability. The numerical solver is based on a multi-block, multigrid, finite-volume method and parallel code for the steady and unsteady Euler and Navier-Stokes equations implemented on overset grids. The Euler algorithm has strictly symmetric characteristics, is capable to capture stationary symmetric vortex flows, and can simulate the flow-instability developments under small asymmetric temporal disturbances. Conical slot suction and blowing with a small amount of mass-flow-rate are introduced to stabilize the originally unstable stationary symmetric vortex flow over a circular-cone and a flat-plate delta wing combination, and to manipulate the vortices over a delta wing symmetrically to increase/decrease normal force and antisymmetrically to produce rolling moment. The theoretical predictions agree well with the Euler computations.

I Introduction

The most interesting phenomena associated with high angle of attack aerodynamics is the sudden onset of vortex asymmetry on the forebody of an air vehicle in symmetric flight. One of the first observations of vortex asymmetry onset was reported in 1951 by Allen and Perkins.¹ Interest in the phenomenon has been intensified since the late 1970’s as concepts for highly maneuverable aircraft have been developed. These high-performance aircraft are expected to operate routinely at angles of attack at which vortex asymmetric is known to occur. When vortex asymmetry occurs, the aerodynamic, stability, and control characteristics of the vehicle change dramatically. In addition, the conventional aerodynamic controls may become ineffective due to the vortex wakes generated by the forebody.

High-angle-of-attack flow control is most effective when applied at the region close to the pointed tip of the forebody. The presence of two closely-spaced vortices around the forebody at high angles of attack enhances the effectiveness. Compared with the wings, control on the forebody is required over a much smaller area and thus physical requirements such as size and weight should be much smaller. The lengthy forebody of a modern fighter further enhances the control effectiveness by providing a long moment arm. Excellent reviews of this activity can be found in papers by Malcolm²,³ and Williams.⁴

One of the devices to suppress the flow asymmetry is the horizontal nose strakes. Coe, Chambers and Letko⁵ showed by wind tunnel tests the alleviating effects of horizontal and symmetrical strakes placed close to the apex of a slender ogive forebody and a slender circular cone. Champigny⁶ gave in his figure 29 that when the narrow wings are located very far forward on the pointed-nose body of an ONERA-S3MA model, the side forces at high angles of attack are substantially reduced. Erickson and Lorincz⁷ shown by water tunnel visualizations that a F-18-type configuration at high angles of attack and zero sideslip are strongly entrained onto the wing by the powerful vortices shed from the wing leading-edge extensions. Nelson and Malcolm⁸ gave in their figure 32 the fluorescent minituft and the smoke/laser sheet photograph of the surface and wake flow around a high performance aircraft model at 20° angle of attack. It is observed that the effect of the strakes is to produce

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a well-defined point of separation at the leading edges of the strakes which results in a symmetrical flow field at a wide range of high angles of attack.

Another promising approaches to forebody vortex-asymmetry management is pneumatic control, i.e. suction/blowing through holes or slots near the forebody tip. A prime potential source for improved control power is the vortex flowfield existing on typical fighter aircraft forebodies. Peake, Owen, and Johnson made wind-tunnel investigations using a slightly blunted 5° semi-angle cone and a 3.5 calibre tangent ogive (16° semi-angle) in a Mach 0.6 airstream. Small amount of air injected either normally or tangentially to the body surface, but on one side of the leeward meridian and beneath the vortex farthest from the wall, were effective in biasing the asymmetry. This phenomenon could be obtained either by changing the blowing rate at constant incidence or by changing incidence at constant blowing rate. Normal injection appeared more effective than tangential injection. With this reorientation of the forebody vortices, the amplitude of the side force could be reduced through zero to reverse its direction. An alternative to blowing from jets is to either blow or suck from a longitudinal slot along the forebody sides. Ng and Malcolm performed a water tunnel test of the F/A-18 model. Suction was in the form of a slot along the 135° windward meridian. At $\alpha = 30°$ and 50° and no sideslip, a very low suction rate was needed to effect the maximum asymmetry in the vortex flow pattern for the respective angles of attack. Most of the pneumatic control methods applied on the forebody fuselage work on the basis of the fuselage boundary layer separation control as pointed out by Malcolm.

As the apex portion of any slender pointed body is nearly a conical body, high angle-of-attack flow over slender conical wing-body combinations were studied in a recent paper using the analytic method developed by Cai, Liu and Luo in Reference 12. The theoretical analysis is based on an eigenvalue analysis on the motion of the vortices under small perturbations, which pertains to the absolute-type of instability. It was noted that the horizontal nose strakes suppress the vortex asymmetry by stretching the two symmetric concentrated vortices away from each other to alleviate the interaction of the two vortices. The theoretical results agree well with available experimental observations and have been corroborated by numerical computations of a three-dimensional time-accurate Euler solver. The Euler solver is based on a multi-block, multigrid, finite-volume method and parallel code for steady and unsteady Euler and Navier-Stokes equations and incorporated with an overset-grid technique.

Incorporating the use of the nose strakes to suppress inherent asymmetry, air suction through symmetric longitudinal slots along rays on the upper surfaces and near the roots of the strakes may benefit from the substantial leverage to be derived from controlling the vortex positions and their interactions. One aim of the present paper is to explore this hybrid technique of the passive strakes and active air suction controls to further stabilize the symmetric vortex pairs at higher angles of attack. The other aim is to study symmetric suction or blowing to increase or decrease the normal force and anti-symmetric blowing and suction to produce rolling moment acting over a slender delta wing.

In the following sections, the theoretical method is summarized and incorporated with appropriate modifications for pneumatic controls and vortex core effects and applied to a slender wing-body combination and a slender wing with slot sources on the upper surfaces. An Euler method is then described and implemented to compute the stationary vortex flowfields and study their stability under small temporal perturbations with and without blowing/suction control for the typical cases studied by the theoretical method. The computational results are compared with the theoretical predictions and lastly some conclusions are drawn.

II Theoretical Method

In this section, the theoretical vortex-flow model and the stability analysis method developed in Refs. 12, 14 and 11 are summarized. Sources are added to the model to simulate the pneumatic control. Semi-empirical modifications to the model are made to account for the effects of the jet-like axial flow of the vortex core.

A The Vortex Velocity Expression

Consider the flow past a slender conical wing-body combination at an angle of attack $\alpha$ and sideslip angle $\beta$ as shown in Fig. 1. The velocity of the free-stream flow is $U_\infty$. The combination has a slender triangular flat-plate wing passing through the longitudinal axis of a cone body. The flat-plate wing has a half vertex angle of $\epsilon$. In a cross-sectional plane at $z$, the wing has a half span $s$, and the center body has a half span $b$. The wing-body combination is assumed to be conical and of infinite length. No effects of trailing edge or body base are considered. At high angles of attack, a pair of symmetric or asymmetric vortices emerges from the sharp leading edges of the wing. To study the stability of the vortex flow, the theoretical model used here is mainly that of Legendre. The vortex model consists of one pair of concentrated vortices separated from the leading edge of the flat-plate wing as shown in Fig. 1. The distributed vortex sheets that connect the leading edges and the two concentrated vortices are neglected since their strength is in general much smaller than that of the two concentrated vortices. The two concentrated vortices are assumed to be conical rays from the body apex $O$. Secondary separation vortices, if any, are weak and thus also neglected. Vortex break-
down is not considered. The flow is assumed to be steady, inviscid, incompressible, conical, and slender. The flow is irrotational except at the center of the isolated vortices.

The governing equation for the velocity potential is the three-dimensional Laplace equation with zero normal flow velocity on smooth body surfaces, and Kutta conditions at sharp edges as boundary conditions. By the principle of superposition, the flow around the body can be obtained by solving the following two flow problems:

Flow problem 1: The flow due to the normal components of the freestream velocity, $U_x = U_\infty \cos \beta \sin \alpha$ and $U_y = U_\infty \sin \beta$.

Flow problem 2: The flow due to the axial component of the freestream velocity, $U_z = U_\infty \cos \beta \cos \alpha$.

Both subject to the boundary conditions. The first flow problem is solved by a conformal mapping $\zeta = \zeta(Z)$ that maps the body contour in the plane $Z = x + iy$ to a circle of radius $r$ in an uniform flow of velocity $(U_x/2, U_y/2)$ in the plane $\zeta = \xi + i\eta$. The second problem is solved by the condition of conical flow in which the flow is invariant along rays emanating from the apex. Details can be found in Ref. 12.

In this paper, the theoretical flow model is modified to account for the pneumatic flow control and the vortex-core effects. For the pneumatic control, a set of sources of strength $Q_{cj}$ is distributed on the body contour at the points $\zeta_{cj}$, where $j = 1, \ldots, N_c$. For the vortex-core effects, a line sink of strength, $Q_c$ is added to each line vortex $\Gamma$, and in the meantime the freestream velocity component along the body axis, $U_z$ used in the second flow problem is augmented by a factor $(1 + K_c)$ where $(K_c > 0)$. $Q_c$ and $K_c$ are related to the strength of the vortex considered, $\Gamma$ by a semi-empirical method. $Q_c = -q_c \Gamma$, and $K_c = \kappa (\Gamma/(2\pi s U_e))^2$ where $q_c = 0.02$, and $\kappa = 0.3$. Detail of the derivations and verifications of the modified model will be presented in Ref. 16.

Let $Z_1$ (or $\zeta_1$ on the transformed plane) be the location of the first vortex. Following Reference 14 the complex velocity at $Z_1$ is as follows.

$$u_1 - iv_1 = \left[ \frac{1}{2} \left( U_n - \frac{U_n r^2}{\zeta_1^2} \right) + \frac{i\Gamma_1}{2\pi} \left( -\frac{1}{\zeta_1 - r^2/\zeta_1} \right) \right]$$

$$- \frac{i\Gamma_2}{2\pi} \left( \frac{1}{\zeta_1 - \zeta_2} - \frac{1}{\zeta_1 - r^2/\zeta_2} \right)$$

$$+ \sum_{j=1}^{N_c} \frac{Q_{cj}}{2\pi} \left( \frac{1}{\zeta_1 - \zeta_j} + \frac{1}{\zeta_1 - r^2/\zeta_j} - \frac{1}{\zeta_1} \right)$$

$$- q_c \Gamma_2 \left( \frac{1}{\zeta_1 - \zeta_2} \left( \frac{d^2 \zeta}{d\zeta^2} \right) \right)$$

$$- \frac{i - q_c}{4\pi} \Gamma_1 \left( \frac{d^2 \zeta}{d\zeta^2} \right)^2$$

$$- \frac{(1 + K_c) U_z Z_1}{s K} + \sum_{j=1}^{N} \frac{Q_j}{Z_1 - Z_j}$$

(1)

where the overbar denotes complex conjugate; $U_n = U_x(1 + iK_x)$; $K = \tan \alpha/\tan \epsilon$ is the Sychev similarity parameter; $K_S = \tan \beta/\sin \alpha$ is the sideslip similarity parameter. The last two terms on the righthand side are the solutions of the flow problem 2, and the other terms are the solution of the first flow problem. $Q_j (j = 1, 2, \ldots, N)$ are the strengths of the point sources at the points $Z_j$ to be determined by $N$ simultaneous equations of the boundary conditions at $N$ points on the body contour in the augmented axial flow. The subscript 1 denotes the values at vortex point $Z_1$ (or $\zeta_1$). A similar expression is obtained for the complex velocity at the center of the other vortex denoted by $Z_2$ (or $\zeta_2$).

It is noted that $U_x$ is the only parameter having the velocity dimension emerging on the right-hand side of the vortex velocity expression, and thus $U_x$ is chosen for the normalization purpose in this analyses.

The sectional source strength, $Q_{c,j}$, is normalized as $C_Q = Q_{c,j} / (s U_x)$. The sectional normal force, $N_{c,j}$, is normalized as $C_x = N_{c,j} / (s \rho_\infty U_x^2 / 2)$. The non-dimensional sectional rolling moment, $l_1$, is normalized as $C_l = l_1 / (s^2 \rho_\infty U_x^2 / 2)$.

Only vortex configurations (locations and strengths of the vortices) that result in zero flow velocities at the two vortex centers can exist in a steady flow. These locations of the vortices are called the stationary positions. The stationary positions $Z_1$ and $Z_2$, and strengths $\Gamma_1$ and $\Gamma_2$ of the two vortices in the above model are determined by solving a set of four algebraic equations by a Newton iteration method. Among the four equations, two equations set the vortex velocities to be zero, and the other two equations set the flow velocities at the separation points to be zero or finite.
B Stability under Small Perturbations

When a vortex pair is slightly perturbed from their stationary positions and then released, its motion follows the vortex velocity. The increments of its coordinates as function of time are governed by a system of two linear homogeneous first-order ordinary differential equations. Define the Jacobian and divergence of the vortex velocity field \( \mathbf{q} = (u, v) \),

\[
J = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix}, \quad D = \nabla \cdot \mathbf{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}
\]

(2)

It is shown\(^\text{12}\) that the eigenvalues of this problem are

\[
\lambda_{1,2} = \frac{1}{2} \left( D_0 \pm \sqrt{D_0^2 - 4J_0} \right)
\]

(3)

where the subscript \( \cdot \) denotes values at the stationary position of the considered vortex.

The eigenvalues \( \lambda_1 \) and \( \lambda_2 \) depend on the Sychev similarity parameter \( K \), the sideslip similarity parameter \( K_S \), and other geometric parameters, e.g., the body-wing span ratio \( \gamma = b/s \). Any perturbation of the stationary positions of the vortex pair can be decomposed into a symmetric perturbation and an anti-symmetric perturbation. The maximum real part of the two eigenvalues \( \lambda_1 \) and \( \lambda_2 \) for each vortex of the stationary vortex pair under small symmetric or anti-symmetric perturbations is used to determine stability in this analysis. A positive value of this variable means perturbation growth (unstable), a negative value means perturbation decay (stable), and a zero value means perturbation remain ( neutrally stable). A vortex pair is stable if and only if both vortices are stable under both symmetric and anti-symmetric perturbations.

III Numerical Method
A The Euler Code

To verify the above theoretical analysis, a three-dimensional, time-accurate Euler solver is used to compute the separation vortex flows over slender conical bodies studied by the theoretical method. The present solver is based on a multi-block, multigrid, finite-volume method and parallel code for the steady and unsteady Euler and Navier-Stokes equations. The method uses central differencing with a blend of second- and fourth-order artificial dissipation and explicit Runge-Kutta type time marching. The resulting code preserves symmetry. Given symmetric initial and boundary conditions, the computations using the steady-flow mode of the Euler solver yield a stationary symmetric vortex-flow solution with an 11 or higher orders of magnitude reduction in the maximum residual. To investigate the stability of the stationary symmetric/asymmetric vortex flow solutions, the steady solution is used as a new initial condition and the time-accurate Euler solver is used to simulate the flow evolution under a small temporal asymmetric perturbations. The unsteady time-accurate computations are achieved by using a 2nd-order accurate implicit scheme with dual-time stepping. The solver has been validated for a number of steady and unsteady cases.\(^\text{18-21}\)

A newly developed overset-grid technique\(^\text{22}\) is implemented to facilitate the grid refinement in the domain of high vorticity.

B Conical Grids

Numerical experiments with Euler code in Reference 13 show that the conical flow assumption of the theoretical model is a good approximation for slender conical bodies at low speeds and therefore a conical grid can be used in the slender conical vortex flow computations. To resolve a nearly conical vortex flowfield by a conical grid, few grid lines are needed in the longitudinal direction. However, much finer grid lines in radial and circumferential directions are required. An extraordinarily fine grid in the cross-flow plane is needed to resolve the high vorticity regions and simulate the vortex interactions and flow instability.

The conical body is infinitely long, while the computational flowfield is finite. Fig. 2 shows the conical grid used in the computations of a wing-body combination of a flat-plate delta wing and a circular-cone body, where part (a) is the grid on the incidence plane, the upstream edge is at \( z = 0 \), and part (b) is the grid on the half exit plane, \( z = 7.11s \), and the grid is bounded by a circle of radius, 25s, where \( s \) is the local semi-span. Only every 4th line is shown in the radial and circumferential directions for clarity. The grid is symmetric with respect to the incidence plane. Every point, except the vertex point, on the lateral boundary of the conical grid has a distance about 25s from the
body. Therefore, the lateral boundary of the conical grid is a far-field boundary.

A close-up view of the conical grid on the exit plane is shown in Fig. 3. Only every 4th line is plotted in the figure for clarity. The grid consists of three layers: the inner layer grid is 5 × 177 × 581 along the longitudinal, radial, and circumferential directions, consisting of 8 blocks, the intermediate layer grid is 5 × 49 × 385 consisting of 2 blocks and the outer layer grid is 5 × 49 × 257 consisting of 2 blocks. This grid is much finer in the radial and circumferential directions than the grids 59 × 50 × 120 and 40 × 65 × 145 used for the full-space Navier-Stokes flow computations of ogive-cylinder body by Degani and Hartwich et al., respectively. The high density of this grid is needed to resolve the high vorticity regions and to simulate the vortex interactions. The computing time for one iteration in double (64 bit) precision is about one second on an 8-processor parallel clusterl computer consisting of AMD Athlon XP1600+CPUs.

The conical grid used for the flat-plate delta wing computations is also an overset grid. The entire grid is symmetric with respect to the incidence plane and consisting of two layers. The inner layer grid is 5 × 97 × 577, and the outer layer grid is 5 × 97 × 385 along the longitudinal, radial, and circumferential directions, respectively. Figure 4 shows the close-up view of the conical grid on the exit boundary plane. Both inner and outer layer grids are decomposed into eight blocks with different density distributions designed to match the local flow gradients and to facilitate the parallel processing.

The freestream Mach number is set at 0.1 to approximate an incompressible flow for the computations reported in the present paper. The boundary conditions on the body surfaces are that the velocity component normal to the surface is zero. Kutta condition at the sharp leading edges of the wing is satisfied automatically with the Euler code. Characteristic-based conditions are used on the upstream boundary of the grid. On the downstream boundary, all flow variables are extrapolated.

C Temporal Asymmetric Perturbations

Small temporal asymmetric perturbations are applied to stationary symmetric/asymmetric Euler solutions as instability triggers in the time-accurate computation of the flow evolution. Asymmetric perturbations are applied because they are found to be most unstable mode in the theory. The small asymmetric perturbations are temporal blowing and suction on the right and left sides of the wing, respectively. The blowing/suction slots on the upper surfaces of the wing are the narrow conical regions bounded by two rays located approximately beneath the vortex cores. The two slots are located symmetrically with respect to the incidence plane. The perturbations are activated in the first time period 0 < t < 1 of the time-accurate Euler computation, where t is a non-dimensional time. In the Euler computations, the blowing/suction velocity, \( V_j \) is a function of \( y/s \), and \( t \),

\[
V_j = \begin{cases} 
V_0 \sin\left(\frac{2\pi t}{\b - y_1}ight) \sin(\pi t) & 0 \leq t \leq 1, y_1 \leq y \leq y_2 \\
0 & \text{otherwise}
\end{cases}
\]  

(4)

The boundary conditions on the suction and blowing area on the upper surfaces of the wing are that the outward normal velocity equals to \(-V_j\) and \(V_j\) respectively.

To calculate the stationary symmetric flow, the Euler code is run in its steady-state mode. Computations are performed starting from a uniform freestream flow until the maximum residual of the continuity equation is reduced by more than 11 orders of magnitude. Such a stringent convergence criterion is needed for stability studies of high angle-of-attack flows as is pointed out.
Fig. 5 Positions of the stationary symmetric vortex pairs over a wing-body combination vs. \( K \), \( \gamma = 0.7 \), by analytic method; and pressure contours in a cross-flow plane, \( \epsilon = 8^\circ \), \( \alpha = 30^\circ \), \( K = 4.108 \), \( C_Q = 0 \), by Euler computation.

by Siclari and Marconi.\textsuperscript{25} In the time-accurate Euler computation, 50 real time-steps are taken in every unit increment of \( t \), and for each real time step the pseudo-steady-flow computation is carried out until four or higher orders of magnitude decline in the maximum residual.

D Steady Slot Blowing or Suction

Small steady narrow-slot blowing and suction normal to the body surfaces are used to control the vortex flow. In the Euler computations, the normal blowing velocity \( V_q \) from a slot between \( y_1 \) and \( y_2 \) on the wing is a function of \( y/s \).

\[
V_q = \begin{cases} 
V_0 \sin\left(\frac{y-y_1}{y_2-y_1}\pi\right) & y_1 \leq y \leq y_2 \\
0 & \text{otherwise}
\end{cases}
\]  

The sectional volume-flow-rate through the blowing slot \( m = 2V_0(y_2 - y_1)/\pi \) is set to be equal to the corresponding sectional source strength, \( Q_c \). When the normal blowing slot is located on the surface of the circular cone, a similar treatment holds.

IV Flat-Plate Delta Wing and Circular Cone Combination

The stability of the stationary symmetric vortex pairs over a combination of a flat-plate delta wing and a circular-cone body at high angles of attack and zero sideslip with and without slot suction/blowing control are studied by the theoretical methods. The theoretical predictions are then verified by the Euler computations for typical cases. The body-width-to-wing-span ratio \( \gamma = 0.7 \).

A No Suction/Blowing Control

In this subsection, no steady suction and blowing is applied, i.e. \( C_Q = 0 \). The theoretical analyses give the stationary symmetric vortex positions versus \( K \) for

\[
K = 1 \sim 6 \text{ with an increment } \Delta K = 1 \text{ as shown by the circles in Fig. 5. To check theoretical results, an Euler computation is performed.}
\]

The Euler computations are performed for the case \( \epsilon = 8^\circ \) and \( \alpha = 30^\circ \). In this case, \( K = 4.108 \). A stationary vortex flow is searched for by running the Euler solver in its steady mode. For this case, however, the theory predicts two possible stationary solutions,\textsuperscript{11} one symmetric and one asymmetric. To obtained the symmetric solution, one expects a symmetric and highly accurate numerical method. The Euler solver used in the present study is based on an explicit central-differencing-type finite-volume scheme. The algorithm is intrinsically symmetric. With a uniform free stream flow as the initial solution and on the three-layer very fine grid, the computations are run in double precision until the maximum residual is reduced by 11 orders of magnitude. The resulting steady-state solution is found to be indeed symmetric with respect to the incidence plane. The computed pressure contours on the cross-flow plane are given in Fig. 5. The computed centers of the two vortex cores are symmetrically located at \( x/s = 0.649 \), \( y/s = \pm 0.912 \) as shown in the figure. The computed centers of the vortex cores for \( K = 4.108 \) are very close to the circular symbol of \( K = 4.0 \) of the analytical results. Fig. 6 gives the computed contours of the longitudinal velocity component in four cross-flow planes along the body axis. It is clearly seen that the vortex sheets are separated from the sharp leading edges and rolling into the vortex cores. Clearly, this solution represents a stationary symmetric vortex flow, which is subjected to a stability examination.

The stability of the stationary symmetric vortex flow is analyzed by the theoretical methods. Figure 7 shows the maximum real part of the eigenvalues for the stability of the symmetric vortex pairs under small symmetric and anti-symmetric perturbations versus
Fig. 7 Maximum real part of eigenvalues of the stationary symmetric vortex pairs over a wing-body combination vs. $K$, $\gamma = 0.7$, $C_Q = 0$.

$K$. The stationary symmetric vortex pair is stable when $K \leq 3.55$ and unstable otherwise. For an isolated slender circular cone, the stationary symmetric vortex pair is always unstable as shown in Reference 12. Thus, by mounting a pair of strakes (i.e. the delta wing) on the circular cone forebody the originally unstable symmetric vortices become stable until $K = 3.55$ for the case $\gamma = 0.7$. In Ref. 13 an Euler computation at $K = 2.3119$ corroborated the theoretical prediction. For the present computational case, $K = 4.108$, Fig. 7 predicts that the stationary symmetric solution is unstable under small perturbations. This is verified by the Euler computations below.

Disturbances of short duration defined by Eq. (4) are introduced to the above symmetric stationary vortex configurations. They consist of suction and blowing at slots close to the wing roots with $y_1 = 0.73s$ and $y_2 = 0.78s$, and $V_0 = 2.07x$. The Euler solver is run in time-accurate mode upon the introduction of the above disturbance. The computation shows that the disturbed flow does not return to its starting stationary configuration even after the initial disturbance has long disappeared. Instead, it wanders farther and farther away until it reaches a new steady-state solution shown in Fig. 8, where the two lines with circles mark the trajectories of the two vortex centers and the contours are constant pressure lines of the newly obtained steady-state solution at the end of the time-accurate computation. Figure 9 shows the vortex core position $x/s$ and $y/s$ versus the nondimensional time $t$. The new steady-state solution is highly asymmetric. The left vortex moved a small distance down toward the wing surface while the right vortex wandered significantly farther above and to the left compared to the original symmetric solution. The center of the left, lower vortex core is at $x/s = 0.353$ and $y/s = -0.914$. The right, upper vortex core center is at $x/s = 1.861$ and $y/s = 0.532$. Figure 10 shows the contours of the longitudinal velocity component in four cross-flow planes along the wing-body combination. Notice that the disturbances are only imposed for a short duration $0 < t < 1$ while the computation is continued without any externally imposed disturbance or asymmetry from $t = 1$ until $t = 50$ (see the abscissa of Fig. 9). This can only be explained by the fact that the initial symmetric vortex solution (obtained under a very stringent convergence criterion) is not a stable configuration and in addition the new asymmetric solution must also be a possible stationary vortex configuration, a fact that is in complete agreement with the theoretical prediction.11

The above computations also demonstrate the symmetric nature of the algorithm and the computer code of the three-dimensional Euler solver used for the present studies. Such symmetry is highly desirable for the study of flow instability. Otherwise, the stationary symmetric vortex flow obtained in Fig. 6 would be elusive because asymmetry in the numerical computa-
Fig. 10 Contours of the longitudinal velocity component in four cross-flow planes of the stationary asymmetric solution triggered by a temporal disturbance over a wing-body combination, \( e = 8^\circ, \gamma = 0.7, \alpha = 30^\circ, K = 4.108, C_Q = 0.0 \).

In order to find the conditions that could supply the needed perturbations to trigger the physical instability and thus drive the flow toward the asymmetric solution.

B Steady Suction/Blowing near Wing Roots

To further stabilize the stationary symmetric vortex pair beyond \( K = 3.55 \) for the strake forebody of \( \gamma = 0.7, \) a pair of point sources of equal strength is positioned symmetrically on the upper surfaces of the wing and near the wing roots. In the vortex velocity expression, Eq. (1), only two control sources are different from zero. They are \( Q_{s1} = Q_{s2} = Q \) at \( Z_{s1} = Z_{s2} = iy_c \). The theoretical analyses give the positions of the stationary symmetric vortex pair under the action of the steady sources versus the source strength coefficient \( C_Q = Q/sU_x \) when \( \gamma = 0.7, y_c/s = \pm 0.75, K = 4.108 \) as shown in Fig. 11. The two symmetric vectors near the wing roots \( V_q \) denote the steady slot suction on the upper surface of the wing. It is seen that the stationary symmetric vortices move upward when the strength of the sources \((C_Q > 0)\) is increased from zero, and downward when the strength of the sinks \((C_Q < 0)\) is increased from zero. It is unexpected that there exists an extraordinary large movement of the vortices when \( C_Q = 0 \sim -0.1 \).

The stability of the stationary symmetric vortices under the steady source action is investigated by the analytical methods. Fig. 12 gives the maximum real part of the eigenvalues of the stationary symmetric vortex pairs versus \( C_Q \) at \( K = 4.108 \). It is seen that when \( C_Q = 0 \) the reading from this figure agrees with that from Fig. 7 as it should be, i.e. the symmetric flow is stable under symmetric perturbations, and unstable under anti-symmetric perturbations. When \( C_Q \neq 0 \), the symmetric vortex pair under anti-symmetric perturbations at first becomes more unstable as \( C_Q \) starts from a positive value (sources) to drop to zero. And even under symmetric perturbations the symmetric flow is also somewhat destabilized in the mean time. However, when \( C_Q \) drops through the critical value \( C_Q = 0.063 \), the originally unstable symmetric vortex flow is abruptly changed to a stable one under anti-symmetric perturbations. And after \( C_Q \) drops through the critical point, the stable state persists for all values of \( C_Q \) and no matter the perturbations are symmetric or anti-symmetric. It is seen that the suppression of the flow asymmetry is achieved through managing the vortex interactions with the wing-body configuration. For the case \( \gamma = 0.7, y_c/s = \pm 0.75, \) and \( K = 4.108, \) the minimum suction rate required to suppress the vortex asymmetry is predicted as \( C_Q = 0.063 \) for each slot by the theory.

To simulate the steady sink in the theoretical analy-
Fig. 13  Vortex center position vs. time after a temporal disturbance is applied to the stationary symmetric vortex-flow solution over a wing-body combination with steady slot suction, $\epsilon = 8^\circ$, $\gamma = 0.7$, $\alpha = 30^\circ$, $C_Q = -0.08$, $y_c/s = \pm 0.75$. The vortex cores are positioned symmetrically with respect to the incidence plane $y = 0$ on the upper surfaces of the wing and near the wing roots. The computational model is $\gamma = 0.7$, $\epsilon = 8^\circ$, and $\alpha = 30^\circ$. The right-hand suction slot is bounded by two rays, $y_1 = 0.738s$ and $y_2 = 0.765s$. The suction velocity $V_q$ is a function of $y/s$ as shown in Eq. (5) where $V_0$ is set equal to $2.327U_\infty$ to match with the non-dimensional sink strength coefficient $C_Q = -0.08$ used in the theoretical analyses. The suction mass-flow-rate coefficient based on the freestream velocity and the wing area, $C_m = 0.02$ for the right-hand side suction slot. The corresponding momentum-flow-rate coefficient $C_p = 0.07$. The same expressions hold for the left-hand side suction slot.

The stationary symmetric flow solution without suction control at $\alpha = 30^\circ$ has been obtained above by the steady-flow Euler solver. A steady-flow Euler computation is performed with the steady suction control activated as given by Eq. (5). Another stationary symmetric flow solution is obtained. Fig. 11 shows the pressure contours over a cross flow plane with the steady suction control. Again, the computed vortex core centers at $C_Q = -0.08$ are very close to the analytical results as shown in Fig. 11. It is seen that the vortex cores move almost vertically downward toward the upper surfaces of the wing due to the suction control as shown in Table 1. In the presence of the central circular-cone body, this movement of the vortex pair is expected to affect the interaction and thus stabilize the vortices as predicted by Fig. 12. To verify this prediction, the stability of the stationary symmetric vortex pair under the steady suction control is computed by the Euler methods.

With the stationary symmetric solution under the steady suction $V_q$ shown by the two arrows near the wing roots in Fig. 11 as initial solution, the temporal suction/blowing $V_j$ perturbation with $y_1 = 0.90s$, $y_2 = 0.95s$, and $V_0 = 2.0U_\infty$ in Eq. (4) is activated in the initial period $0 < t < 1$, and the suction control $V_q$ stays for all the time $t > 0$, the time-accurate Euler solver is used to simulate the flow development. The vortex-core positions versus $t$ are recorded in Fig. 13. It is observed that the temporally perturbed symmetric vortex flow under steady suction control, unlike that without suction control as shown in Figures 9, returns to the initial stationary symmetric flow when $t \geq 16.0$. Therefore, the stationary symmetric vortex pair under persistent suction control with the suction mass-flow-rate coefficient $C_m = 0.02$, or the non-dimensional source strength $C_Q = -0.08$ is indeed stable under small perturbations. This result agrees with the theoretical prediction of Fig. 12 given above. The steady symmetric suction with $C_m = 0.02$ for each slot at $y_c/s = \pm 0.75$ stabilizes the originally unstable symmetric vortex flow over the wing-body combination.

The steady suction not only stabilizes the stationary symmetric vortices, but also enhances the vortex core, and thus increases the vortex lifting force acting on the strakes. It is interesting to compare the various flow parameters with and without the suction control. Table 1 compares the vortex-core center positions $(x/s,y/s)$, and the total velocity $U/U_\infty$, pressure coefficient $C_p$ and Mach number $M$ at the vortex core center with and without the suction control. The static pressure in the vortex core is decreased by 40% due to the steady symmetric slot suction of $C_m = 0.02$ at $y_c/s = \pm 0.75$.

**Table 1  Comparison of flow parameters at the vortex-core center over a wing-body combination of $\epsilon = 8^\circ$, $\gamma = 0.7$, $\alpha = 30^\circ$ with and without steady slot suction at $y_c = \pm 0.75$.**

<table>
<thead>
<tr>
<th>Suction Control</th>
<th>$C_m = 0.0$</th>
<th>$C_m = -0.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x/s$</td>
<td>0.649</td>
<td>0.379</td>
</tr>
<tr>
<td>$y/s$</td>
<td>$\pm 0.912$</td>
<td>$\pm 0.938$</td>
</tr>
<tr>
<td>$U/U_\infty$</td>
<td>2.861</td>
<td>3.214</td>
</tr>
<tr>
<td>$C_p$</td>
<td>$-10.214$</td>
<td>$-13.982$</td>
</tr>
<tr>
<td>$M$</td>
<td>0.289</td>
<td>0.325</td>
</tr>
</tbody>
</table>

C Variation of Suction Positions

To find the optimum position of the steady slot suction to effectively suppress the flow asymmetry, we move the suction slot pair having a constant sink strength over the entire upper surfaces of the wing and the cone. The theoretical analyses give the positions of the stationary symmetric vortex pair versus the location of the suction slot pair $y_c/s$ or $\theta_c$ under the conditions $C_Q = -0.065$ and $K = 4.108$ in Fig. 14. $\theta_c$ is measured from the leeward side of the incidence plane. For clarity, only one side of the symmetric vortices is shown in the figure. When the suction slot pair is located on the wing, only the right-hand vortex
Fig. 14 Position of the stationary symmetric vortex pair over a wing-body combination vs. control sink position, $\gamma = 0.7$, $C_Q = -0.065$, $K = 4.108$.

Fig. 15 Maximum real part of eigenvalues of the stationary symmetric vortex pair over a wing-body combination vs. control sink position, $\gamma = 0.7$, $C_Q = -0.065$ and $K = 4.108$.

positions are shown. When the suction slot pair is located on the cone, only the left-hand vortex positions are shown.

The stability of the stationary symmetric vortex pairs found above is studied by the theory. Fig. 15 gives the maximum real part of the eigenvalues of the stationary symmetric vortex pairs under the same conditions as above. It is seen that when $40.4^\circ < \theta_c < 90.0^\circ$ on the cone surface and 0.700 < $y_s$/s < 0.756 on the wing upper surface, the stationary symmetric vortex pairs are stable under small perturbations, and unstable otherwise. It means that when the suction slot pair is located near the intersection of the wing and cone the small amount of the non-dimensional sink strength, $C_Q = -0.065$ is enough to stabilize the stationary symmetric vortex pair at $K = 4.108$.

For a wing-body combination with $\epsilon = 8^\circ$, when the suction slots are located on the cone surface at $\theta_c = \pm 70^\circ$, the theoretical analyses in Fig. 16 give the minimum value of the non-dimensional sink strength, $C_Q = -0.065$ is enough to stabilize the stationary symmetric vortex pair at $K = 4.108$.

Fig. 16 Minimum sink strength coefficient $C_Q\infty$ vs. $\alpha$ for a wing-body combination, $\gamma = 0.7$, $\epsilon = 8^\circ$, $\theta_c = \pm 70^\circ$; and normal force coefficient $C_x\infty$ vs. $\alpha$ with and without minimum sink slot suction.

coefficient, $C_Q\infty = Q/(sU\infty)$, which is capable to stabilize the stationary symmetric vortex pair, versus angle of attack $\alpha$. It is seen that with $C_Q\infty = -0.13$ the stationary symmetric vortex pair remain stable at an angle of attack up to $52^\circ$. $C_Q\infty = -0.13$ corresponds to $C_m = 0.065$ for each suction slot. The steady slot suction on the upper surface of the body will produce an increment of the normal force on the body. The present analytic method is used to make a rough estimation of the normal force increment. By a pressure integration over the body surface excluding the sinks, the normal force, $N_x$, is obtained for the cases with and without suction. Here the normal forces are normalized as $C_x\infty = N_x/(sU^2/2)$ are shown in the same Figure above. The increment of $C_x\infty$ at $\alpha = 52^\circ$ is 1.0 due to the slot suction. In terms of the conventional aerodynamic force coefficient which is based on the total area of the wing, the increment of the conventional normal force coefficient is 0.5 which is quite significant.

D Steady Suction/Blowing at $\theta_c = \pm 45^\circ$

The stability of the stationary vortex pair with the steady suction/blowing slots located on the upper surface of the circular cone are studied by the theoretical methods. Fig. 17 shows the analytic results of the positions of the stationary symmetric vortex pair over the wing-body combination of $\gamma = 0.7$ versus $C_Q$ when the slots are located at $\theta_c = \pm 45^\circ$ and $K = 4.108$. When strength of the control source increases from zero the vortex pair moves upward and when the strength of the source decreases from zero the vortex pair moves downward.

Fig. 18 plots the analytic results of the maximum real part of the eigenvalues of the stationary symmetric vortex pairs under the same conditions as above. It is seen that when $C_Q < -0.057$ or $C_Q > 0.37$, the stationary symmetric vortex pair is stable under small perturbations, and unstable otherwise. It means that
not only the steady suction can be used to suppress the onset of flow asymmetry, but also the steady blowing is capable to stabilize the symmetric vortex flow if the blowing rate is high enough. However, the suction control is much more efficient than the blowing control because the minimum mass-flow rates required to suppress the flow asymmetry is much less for the suction control. The effects of steady blowing control with the critical rate $C_Q = 0.37$ at $\theta_c = \pm 45^\circ$ found from the above analyses are studied in the next subsection.

E Variation of Steady Blowing Positions with $C_Q = 0.37$

It is found that a steady blowing of $C_Q = 0.37$ at $\theta_c = \pm 45^\circ$ is capable to suppress the flow asymmetry over the wing-cone combination of $\gamma = 0.7$. An analytic survey of the stationary symmetric vortex locations and their stability is made for the blowing slots moving over the entire upper surfaces of the combination body with the constant flow rate, $C_Q = 0.37$ and at $K = 4.108$. Fig. 19 shows the analytic results of the positions of the stationary symmetric vortex pair versus the slot position, $y_c/s$ or $\theta_c$. When the blowing slots move from the leeward symmetry plane to the wing-cone intersection and from the wing root to the wing tip, the vortex pair move inboard.

Fig. 20 plots the maximum real part of the eigenvalues of the stationary symmetric vortex pairs under the same conditions as above. When $0.494 < y_c/s < 0.634$ or $45.0^\circ < \theta_c < 64.9^\circ$, the stationary symmetric vortex pair is stable under small perturbations, and unstable otherwise. It means that the steady blowing slots located in certain region on the upper surface of the circular cone are also capable to suppress the flow asymmetry, though the required mass-flow-rate is rather large.

V Flat-Plate Delta Wing

The stability of the stationary vortex pair separated from the sharp leading edge of a slender flat-plate delta wing at high angles of attack and no sideslip is first studied by the modified theoretical method and
verified by the Euler method. Then symmetric and anti-symmetric slot blowing and suction are applied on the upper surface of the wing to control the aerodynamic normal force and rolling moment acting on the wing, while the leading-edge vortex pair remains stationary and stable.

A No Suction/Blowing Control

Fig. 21 gives the positions of the stationary symmetric vortex pair versus $K$ obtained shown by the circle symbols. They are obtained by the analytic method given in Section of Theoretical Method. No stationary asymmetric vortex pair is found when the wing has no sideslip. Fig. 22 shows the maximum real part of eigenvalues $\lambda_1$ and $\lambda_2$ of the vortex motion for the stationary symmetric vortex pairs versus the similarity parameter $K$. The eigenvalues remain negative for the whole range of $K$ considered. This indicates that the stationary symmetric vortex pair over the flat-plate delta wing is stable for all angles of attack.

The theoretical results are verified by the Euler computation presented in Section of Numerical Method. The computational model is a flat-plate delta wing with $\epsilon = 8^\circ$, $\alpha = 28^\circ$, that is $K = 3.783$. Using the freestream flow as the initial condition and running the Euler code in its steady-flow mode yield a stationary symmetric solution. The computed pressure contour in a cross-flow plane of the wing is shown in Fig. 21. It is seen that the computed center of the vortex core nearly coincides with the circle symbol of $K = 4$ given by the theory. The stability of this stationary symmetric solution has been studied by the Euler method in Reference 13. It was shown that the stationary symmetric solution is indeed stable under small temporal perturbations. Hence the theoretical predictions are confirmed by the Euler computations.

B Steady Slot Blowing or Suction at Root Chord

The slender flat-plate delta wing is considered as a forebody control surface. A steady slot suction or blowing is applied on the upper surface of the wing along its root chord to manage the aerodynamic normal force acting on it. Take $Q_{c1} = Q_{c2} = Q$ and $y_{c1} = y_{c2} = y_c = 0$ in the vortex velocity equation, Eq. (1). The analyses yield the positions of the stationary symmetric vortex pairs versus $C_Q$ shown in Fig. 23 by the circle symbols, where $C_Q = Q/sU_x$. In the figure a vector $V_y$ at the wing root denotes the steady slot suction on the upper surface of the wing. The maximum real part of the eigenvalues of the stationary symmetric vortex pairs versus $C_Q$ are given in Fig. 24. The theory predicts that the stationary symmetric vortex pairs are stable in the range considered.

To verify the above theoretical results, numerical
calculation of steady Euler equation is performed for the case \( \epsilon = 8^\circ, \alpha = 28^\circ \), with a suction slot whose center line is positioned at \( y/s = 0 \) on the upper surfaces of the wing. The suction slot is bounded by two rays, \( y_1 = -0.038s \) and \( y_2 = 0.038s \). The suction velocity \( V_q \) is given by Eq. (5) with \( V_0 = -1.309U_\infty \). The corresponding source strength coefficient is \( C_Q = -0.1 \). The computed pressure contours on a cross-flow plane are shown in the Figure 23. The location of the computed center of pressure contours agrees with that of the analytic result of the circle symbol, \( C_Q = -0.1 \).

With the stationary symmetric solution under the steady suction \( V_q \) shown by the arrow at the wing root in Fig. 23 as initial solution, the temporal suction/blowing \( V_j \) perturbation with \( y_1 = 0.738s \), \( y_2 = 0.788s \), and \( V_0 = 2.0U_\infty \) in Eq. (4) is activated in the initial period \( 0 < t < 1 \), and the suction control \( V_q \) stays for all the time \( t > 0 \), the time-accurate Euler solver is used to simulate the flow development. The vortex-core positions versus \( t \) are recorded in Fig. 25. It is observed that the temporally perturbed symmetric vortex flow under steady suction control returns to the initial stationary symmetric flow when \( t \geq 5.5 \). Therefore, the stationary symmetric vortex pair under the persistent suction control \( C_Q = -0.1 \) at \( y/c = 0 \) is indeed stable under small perturbations. This result agrees with the theoretical prediction of Fig. 24 given above.

To show the effectiveness of the slot suction control in producing normal force, the present analytic method is implemented to make an approximate estimation of the normal force increment due to the control. The normal force is obtained by a pressure integration over the wing from the analytic solution with the source point exempted. The normal force coefficient \( C_{x,\infty} \) and the sink strength coefficient \( C_{Q,\infty} \) are normalized with the freestream velocity \( U_\infty \) rather than its \( x \)-component \( U_x \). Fig. 26 gives \( C_{x,\infty} \) on the wing of \( \epsilon = 8^\circ \) with control sink \( C_{Q,\infty} = -0.05 \) at \( y/c = 0.0 \) and without control sink, and also \( C_Q \) vs. \( \alpha \) at \( C_{Q,\infty} = -0.05 \). For \( C_{Q,\infty} = -0.05 \), the suction mass-flow-rate coefficient \( C_m = 0.025 \) and the momentum-flow-rate coefficient \( C_p = 0.05 \). The increment of the normal force coefficient based on the total wing area due to the slot suction control is about 0.25 and is much greater than the consumed momentum-flow-rate coefficient \( C_p \). Shanks’ low-subsonic measurements for a flat-plate delta wing of sweep angle 82° (or \( \epsilon = 8^\circ \)) showed that an lift-coefficient increment of 0.25 can be obtained by an increment of the angle of attack of 8° before the wing stall.

C  Steady Anti-Symmetric Slot Suction and Blowing

To generate a rolling moment over the flat-plate delta wing, a pair of steady slot suction and blowing is applied on the upper surface of the wing at the slots located anti-symmetrically with respect to the incidence plane of the wing. With the blowing and suction

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Fig. 24  Maximum real part of eigenvalues of the stationary symmetric vortex pairs over a flat-plate delta wing vs. \( C_Q \), \( y/c = 0 \), \( K = 3.783 \).

Fig. 25  Vortex center position vs. time after a temporal disturbance is applied to the symmetric stationary vortex solution over a flat-plate delta wing with steady slot suction, \( \epsilon = 8^\circ, \alpha = 28^\circ \), \( y/c = 0 \), \( C_Q = -0.1 \).

Fig. 26  Normal force coefficient \( C_{x,\infty} \) vs. \( \alpha \) over a flat-plate delta wing with steady slot suction, \( \epsilon = 8^\circ \), with and without \( C_{Q,\infty} = -0.05 \) at \( y/c = 0 \); and \( C_Q \) vs. \( \alpha \) at \( C_{Q,\infty} = -0.05 \).
Fig. 27 Positions of the stationary asymmetric vortex pairs over a flat-plate delta wing with steady anti-symmetric source and sink vs. the slot position $y_c/s$, $C_Q = \pm 0.1$, $K = 3.783$ by analytic method; and pressure contours in a cross-flow plane of the wing, $\epsilon = 8^\circ$, $\alpha = 28^\circ$ ($K = 3.783$), $y_c/s = \pm 0.5$, by Euler computation.

strength coefficient $C_Q = \pm 0.1$ and $K = 3.783$, the positions of the stationary asymmetric vortex pair and their stability versus the slot position $y_c/s$ are given in Figures 27 and 28 respectively by the theoretical method for $0 < y_c/s < 0.99$.

To verify the analytic results, Euler computations are performed for the wing of $\epsilon = 8^\circ$ and $\alpha = 28^\circ$, corresponding to $K = 3.783$ with the two slots located at $y_1 = \pm 0.48s$, and $y_2 = \pm 0.52s$ as shown by the vector $V_q$ in Fig. 27 and $V_0 = \pm 1.844U_\infty$ in Eq. (5), corresponding to $y_c/s = \pm 0.5$ and $C_Q = \pm 0.1$. In Fig. 27 the pressure contours in a cross-plane of the wing are plotted. The computed centers of the pressure contours nearly coincide with the circle symbols of $y_c/s = 0.5$ obtained by the theory.

Fig. 28 shows the maximum real part of the eigen-values for the stability of the stationary asymmetric vortex pairs, under small symmetric and anti-symmetric perturbations. The stationary asymmetric vortex pair is stable when $-0.814 < y_c/s < 0.814$ when $C_Q = \pm 0.1$ and $K = 3.783$.

The theoretical prediction that the stationary asymmetric vortex pair is stable when the source and sink of strength coefficient $C_Q = \pm 0.1$ and located at $y_c/s = \pm 0.5$ is to be verified by the Euler methods. The Euler solver is run again in time-accurate mode upon the introduction of the same temporal disturbance used in the last subsection. Fig. 29 shows the history of the vortex core movement. The disturbed asymmetric vortex pair returns to the initial stationary positions when $t \geq 6.5$. Therefore, the stationary asymmetric vortex pair generated by the steady slot blowing and suction is stable under small perturbations. Therefore the computational result confirms the theoretical prediction of Fig. 28.

The effects of the source and sink strength on the position of the stationary asymmetric vortex pairs and their stability are studied. Fig. 30 gives the theoretical predictions of the vortex positions versus the strength coefficient $C_Q$ for a flat-plate delta wing at $y_c/s = \pm 0.5$ and $K = 3.783$ for $-0.4 < C_Q < 0.4$ by the circle symbols. In this figure, the pressure contours in a cross-flow plane of the wing is plotted for $K = 3.783$. In the computation, suction and blowing is applied anti-symmetrically on the upper surface and the velocity is a function of $y/s$ as shown in Eq. (5), where $y_1 = \pm 0.47s$, $y_1 = \pm 0.53s$ and $V_0 = \pm 2.274U_\infty$ correspond to $y_c/s = \pm 0.5$ and $C_Q = \pm 0.185$. The computed centers of pressure contours are located near to the circle symbols of $C_Q = \pm 0.2$ obtained by the theory.

Fig. 31 shows the maximum real part of the eigenvalues for the stability of the stationary asymmetric vortex pairs under small symmetric and
Fig. 30 Positions of the stationary asymmetric vortex pairs over a flat-plate delta wing with anti-symmetric source and sink vs. the source strength coefficient $C_Q$, $y_c / s = \pm 0.5$, $K = 3.783$, by analytic method; and pressure contours in a cross-flow plane of the wing, $\epsilon = 8^\circ$, $\alpha = 28^\circ$ ($K = 3.783$), $y_c / s = \pm 0.5$, $C_Q = \pm 0.2$ by Euler computation.

Fig. 31 Maximum real part of eigenvalues of the stationary asymmetric vortex pairs over a flat-plate delta wing with anti-symmetric source and sink vs. the control source strength coefficient $C_Q$, $y_c / s = \pm 0.5$, $K = 3.783$.

Fig. 32 Maximum source and sink strength coefficient $C_Q$ and the corresponding rolling moment coefficient $C_l$ and normal force coefficient $C_x$ over a flat-plate delta wing vs. the slot position $\pm y_c / s$, $K = 3.783$.

32 shows the maximum values of the strength of the source and sink coefficient $C_Q$ and the corresponding rolling moment coefficient $C_l$ and normal force coefficient $C_x$ versus their position $y_c / s$ at $K = 3.783$, where $C_l = \text{(total rolling moment)} / (\delta \rho U_s^2 / 2)$. From Fig. 32, the maximum value of $C_Q$ and $C_l$ produced by this value of $C_Q$ increase as the the slots move from the wing tips to its root, while the normal forces remain practically constant. When $y_c / s = \pm 0.5$ and $K = 3.783$, the maximum $C_Q$ is 0.196, which agrees with the reading from Fig. 31, and the corresponding $C_l = 5.2$. For a wing of $\epsilon = 8^\circ$ at $\alpha = 28^\circ$, the conventional rolling moment coefficient normalized with $U_\infty$, the total wing area and the total wing span is 0.29. Corresponding to $C_Q = 0.196$, $C_m = 0.0460$ for each slot. Shanks’ low-subsonic measurements for a flat-plate delta wing of $\epsilon = 8^\circ$ gave $C_l = 0.757$ at $\alpha = 28^\circ$ and $\beta = -5^\circ$. Therefore in this case, the rolling moment produced by the anti-symmetric slot blowing and suction is much larger than that produced by a sideslip of $-5^\circ$ of the wing without blowing and suction.

VI Conclusions

The stability theory presented by Cai, Liu, and Luo,\textsuperscript{12} is further developed to study pneumatic control of vortices over slender conical bodies at high angles of attack and low speeds. The analytic predictions on the positions of stationary vortices and their stability are verified by a three-dimensional time-accurate Euler code for typical cases.

In the present analysis, sources are added to the body surfaces to simulate the pneumatic control, and sink lines are superposed on the vortex lines and the free-stream velocity component in the body axis direction is augmented to account for the effects of the vortex core.

The computational method is based on a multi-block parallel three-dimensional finite-volume method
for the Euler equations on overset grids. Unsteady time-accurate computations are achieved by using a 2nd-order accurate implicit scheme with dual-time stepping. Stationary vortex configurations are first captured by running the Euler code in its steady-state mode. After a stationary vortex flow configuration is obtained, a temporal asymmetric perturbation consisting of suction and blowing of short duration on the left- and right-hand side of the wing is introduced to the flow and the Euler code is run in time-accurate mode to determine if the flow will return to its original undisturbed condition or evolve into a different steady or unsteady solution. The former case indicates that the original stationary vortex configuration is stable while the latter case proves it unstable or neutrally stable.

Three typical examples of slot blowing/suction controls with a mass-flow-rate coefficient less than 0.1 are given by the theoretical method and verified by the Euler computation.

(1) A hybrid technique of the passive strakes and active slot suction on the upper surface near the strake-body intersection suppresses the forebody asymmetry onset till an angle of attack $5^\circ$.

(2) A suction or blowing along the upper surface wing-root chord of a slender flat-plate delta wing produces a normal-force increment equal to that obtained by an angle-of-attack increment of $8^\circ$ in the region of no wing stall.

(3) An anti-symmetric blowing and suction on the upper surface of the slender delta wing produces a rolling moment much greater than that obtained by a sideslip of $5^\circ$ at an angle of attack $28^\circ$.

The theoretical method provides a valuable means to explore the flow control over a slender conical forebody at high angles of attack and low speeds. The Euler computation serves as a check on the theory.

References


