ABSTRACT

The lag model proposed by Olsen and Coakley (2001) is applied to simulate the steady and unsteady transonic flows in a diffuser. The unsteady flows are induced by imposing fluctuating back-pressures. A fully implicit time-accurate multigrid algorithm is used for solving the Navier-Stokes equations and the coupled k-ω turbulence model equations. Two test cases are investigated, one with a weak shock in the channel corresponding to a static-to-total pressure ratio $R_p=0.82$ at the diffuser exit and the other with a strong shock corresponding to $R_p=0.72$. The results with the implementation of the lag model are closer to the experimental data for the strong-shock case. However, the computational results are almost the same with and without the lag model for the weak-shock case.

1. INTRODUCTION

The understanding and prediction of unsteady flows are an important capability in the analysis and design of modern aircraft and the turbomachinery of gas-turbine engines. The major improvement in computing power and computational methods has seen the appearance of unsteady flow computations that employ Euler and Navier-Stokes equation models. In spite of these recent advances, unsteady flow with shock interaction remains computationally expensive. Moreover, appropriate turbulence models for such problems must be used to ensure accuracy.

Significant progress has been made in time dependent algorithms to decrease computational demands. In addition the coupling of the turbulence model must be included. Jameson first proposed an efficient multigrid driven implicit approach to the solution of the Euler equations. It uses central differences in space and an implicit multistep discretization in time; a large set of simultaneous non-linear equations is formed and marched to steady state in pseudo-time through a multigrid algorithm within each real time step. This approach has been exploited for aero-elastic studies and for unsteady flows in turbomachinery (see Liu et al., 2001 and Yao et al. 2001). Of note is that this method also facilitates the incorporation of turbulence models; the flow and turbulence model equations are not treated separately and hence can be fully converged at any time step.

A fully implicit time-accurate multigrid algorithm is developed for solving the Navier-Stokes equations, (Liu and Ji, 1996). The scheme eliminates the CFL stability limit by using implicit time-accurate discretization while the time-required at each time step is small and comparable with that of an explicit time-marching scheme. Local time stepping, residual smoothing and multigrid techniques are also used to accelerate the convergence for the turbulence model equations through the introduction of a pseudo-time-marching problem. Liu & Ji showed that this was essential for good convergent characteristics.

The k-ω two-equation turbulence model cannot account for non-equilibrium effects, such as those encountered in large pressure gradients involving separation and shock waves as found in nozzles and diffusers. More complex Reynolds stress models address this problem, but extensive studies show difficulties for their practical use. They are computationally more involved and numerically stiff, hence generally not used. Non-linear algebraic Reynolds stress (ARS) models, has emerged as a simpler alternative. Notwithstanding the problems of a full Reynolds stress formalism, it is known that such models also respond overly rapidly to mean flow conditions as one- and two-equation models do. Olsen & Coakley (2001) recently proposed a new class of models, which is termed a lag model. The basic idea of the lag model is to take a baseline two-equation model
and couple it with a third equation, the lag equation, to model the non-equilibrium effects for the eddy viscosity. Its simplicity is an added advantage and thus it serves as an alternative to the algebraic stress model for three-dimensional flows.

This paper describes the use of this lag model in the context of the implicit multigrid driven algorithm for the unsteady transonic diffuser flow induced by the backpressure fluctuations. The system of flow and turbulence equations is fully coupled and a fully converged solution is achieved at each time advancement. There are no stability constraints on the implicit time step, and the size is determined purely from the standpoint of the flow physics of the problem. In problems where the frequency of the oscillation is low, big time steps can be used to advantage.

In problems where the frequency of the oscillation is high, small time steps are needed, and the implicit multigrid driven algorithm is the only viable option. In other problems where the frequency of the oscillation is moderate, smaller time steps can be used to advantage.

A simple configuration but computationally challenging problem is the unsteady flow due to fluctuating back-pressure in a transonic diffuser. Boger et al. (1983) Salmon et al. (1983), and Sajben et al. (1984) presented experimental measurements for the pressure field in a transonic and supersonic diffuser with an oscillating shock wave. Liu and Cокley (1984) numerically investigated this configuration using a modified MacCormack’s hybrid method for Navier-Stokes equation and k-ω turbulent models. In this paper calculations are conducted of the same test problem to assess the veracity of the lag model for unsteady problems.

In the following sections, the mathematical model and the numerical solution of this problem are outlined briefly. This is followed by the discussion of the computed numerical results for the flow field. Conclusions will be made in the final section.

2. MATHEMATICAL MODELING

In this section, the mathematical model and the numerical solution method are briefly outlined. A more detailed description of the numerical method used in this modeling work is described in Liu and Ji. (1996)

Governing Equations

The physics of the unsteady compressible turbulent flow with the k-ω and lag model can be expressed by the equations as follows:

\[
\frac{\partial}{\partial t} (\rho E) + \frac{\partial}{\partial x_j} (\rho u_j H) = \frac{\partial}{\partial x_j} \left[ u_j \frac{\partial}{\partial x_j} (\rho u_j H) \right] - q_j
\]

Turbulent mixing energy:
\[
\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_j} (\rho u_j \omega)
\]

Specific dissipation rate:
\[
\frac{\partial}{\partial t} (\rho \omega) + \frac{\partial}{\partial x_j} (\rho u_j \omega)
\]

Turbulent eddy viscosity:
\[
\frac{\partial}{\partial t} (\rho \nu_t) + \frac{\partial}{\partial x_j} (\rho u_j \nu_t)
\]

where \( t \) is time, \( x_j \) position vector, \( \rho \) is the density, \( u_j \) velocity vector, \( p \) pressure, \( \mu \) dynamic molecular viscosity, \( \nu_{\text{ke}} \) kinematic equilibrium turbulent eddy viscosity, \( k \) turbulent mixing energy, and \( \omega \) specific dissipation rate. The total energy and enthalpy are \( E = e + k + u_j u_j / 2 \) and \( H = h + k + u_j u_j / 2 \), respectively, with \( h = e + p / \rho \) and \( e = p / (\gamma - 1) \rho \).

The term \( \gamma \) is the ratio of specific heats. Other quantities are defined in the following equations:

\[
\mu_t = \rho \nu_t
\]

\[
\nu_{\text{ke}} = e^\gamma k / \omega
\]

\[
R_t = \rho k / \mu \omega
\]

\[
S_y = \frac{1}{2} \left( \frac{\partial u_j}{\partial x_j} + \frac{\partial u_i}{\partial x_i} \right)
\]

\[
\tau_y = 2 \mu_t (S_y - \frac{1}{3} \frac{\partial u_i}{\partial x_i} \delta_{ij}) - \frac{2}{3} \rho k \delta_{ij}
\]

\[
\tau_y = 2 \mu_t (S_y - \frac{1}{3} \frac{\partial u_i}{\partial x_i} \delta_{ij}) + \tau_{ij}
\]

\[
q_j = -\left( \frac{\mu}{\mu_{\text{ke}}} + \frac{\mu_t}{\mu_{t_{\text{ke}}}} \right) \frac{\partial H}{\partial x_j}
\]

\[
a(R_t) = a_0 \left( \frac{R_t + R_{\text{ke}}}{R_t + R_{\text{ke}}} \right)
\]
Where Pr$_{t}$ and Pr$_{r}$ are the laminar and turbulent Prandtl numbers, respectively. The model is as proposed by Wilcox (1988) which does not require wall distance information. The other coefficients are:

\[ a_0 = 0.35, \quad R_{T_0} = 1, \quad R_{x_0} = 0.01, \quad \varepsilon = 5/9, \quad \varepsilon^* = 1, \quad \beta = 0.075, \quad \beta^* = 0.09, \quad \sigma = 0.5, \quad \sigma^* = 0.5 \]

In contrast to a standard k-$\omega$ turbulent model simulation, an additional equation, the so-called lag model, is included here, which is intended to better account the behavior of flows with separation.

Numerical Methods
To solve the equations described in the last section, the integral forms of the conservation equations are discretized on quadrilateral cells using the finite volume method. A staggered scheme is used for the coupling discretized in space, the governing equations are reduced to a set of ordinary differential equations with only derivatives in time, which can be solved using a multi-stage Runge-Kutta type scheme. To accelerate the convergence, unsteady multigrid method proposed by Jameson (1991) and further implemented by Liu and Ji (1996) is applied in the present study for all 7 equations. The basic idea can be summarized as follows:

The governing equation after the space discretization, can be written as:

\[ \frac{d\tilde{W}}{dt^*} + \tilde{R}^*(\tilde{W}) = 0 \]

where $t^*$ is a pseudo-time variables and $\tilde{R}^*(\tilde{W})$ is the vector of the unsteady residuals. For the second order fully implicit scheme, $\tilde{R}^*(\tilde{W})$ can be expressed as:

\[ \tilde{R}^*(\tilde{W}) = \tilde{R}(\tilde{W}) + (3/2\Delta t)(\tilde{W}V^{s+1}) - (2/\Delta t)(\tilde{W}^{s+1}V^s) + (1/2\Delta t)(\tilde{W}^{s-1}V^{s-1}) \]

where $V$ is the volume of a computational cell.

The m-stage Runge-Kutta time-stepping scheme for the solution of the above pseudo-time problem reads:

\[ \tilde{W}^{n+1,0} = \tilde{W}^n \]
\[ \tilde{W}^{n+1,q} = \tilde{W}^n - \alpha_q \Delta t \tilde{R}^* \tilde{W}^{(n+1,q-1)} \]
\[ \tilde{W}^{n+1} = \tilde{W}^{(n+1,m)} \]

3. RESULTS AND DISCUSSIONS
The numerical methods presented above are applied to a 2-D steady and unsteady convergent/divergent diffuser investigated by Sajben et al. (1984). The bottom wall is flat and the geometry of the upper wall is given by;

\[ \tilde{h}(\tilde{x}) = \frac{\alpha \cosh \zeta}{(\alpha - 1) + \cosh \zeta} \]

where

\[ \zeta = \frac{C_1(\tilde{x}/\tilde{l}) + C_2(\tilde{x}/\tilde{l})^3}{(1-\tilde{x}/\tilde{l})^{1/2}} \]

the various constants for the top wall are given in the following table. The height of the throat is $h_{th}=1.7322$ inches.

<table>
<thead>
<tr>
<th>Table 1 Constants for channel height</th>
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<tbody>
<tr>
<td><strong>Constant</strong></td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\tilde{l}$</td>
</tr>
<tr>
<td>$C_1$</td>
</tr>
<tr>
<td>$C_2$</td>
</tr>
<tr>
<td>$C_3$</td>
</tr>
<tr>
<td>$C_4$</td>
</tr>
</tbody>
</table>

3
The key parameter which characterizes this diffuser flow is the pressure ratio $R_p = p_2 / p_{1t}$, which is defined as the static pressure at the outlet divided by the inlet total pressure. The flow in the diffuser can be classified into the weak-shock and the strong-shock categories based on the pressure ratio. For the latter, there is shock-induced flow separation, while no flow separation occurs for the weak-shock flow type. Two cases are investigated in the present work, one weak-shock case and one strong-shock case corresponding to $R_p=0.82$ and $R_p=0.72$, respectively. The Reynolds number based on the channel width is $7 \times 10^5$ for both the strong-shock and the weak-shock cases.

Boundary conditions are specified as follows:

At the inflow boundary:

$$p_{1t} = p_{10} = 19.58 \text{ psia}, T_{1t} = 500 \text{R}, M_1 = 0.46,$$

where $p_{1}$ is the total pressure, $T_{1}$ the total temperature, $M$ is the Mach number.

At the outflow boundary:

Only the pressure is specified and all the other variables are extrapolated. To simulate the unsteady flow, the outlet boundary pressure condition is imposed by:

$$P_2 = P'_e + P'_e \sin(2\pi ft)$$

where $P'_e$ is the amplitude of the oscillation, $P'_e / P_e = 0.01$ and $f$ is the frequency.

Only $f=75 \text{ hz}$ is tested in the present study.

On the wall, zero velocity is imposed and the pressure is extrapolated to the wall. The turbulent mixing energy $k$ is set to zero. The specific dissipation rate $\omega$ must satisfy the following asymptotic solution as the wall is approached:

$$\omega \rightarrow \frac{6\nu_w}{\beta y^2}$$

as the distance $y \rightarrow 0$

In all of our numerical examples, the above equation is enforced only at the first grid point from the wall.

Fig. 2 shows the grid distribution used in the present study based on $321 \times 65$. The grid is generated algebraically. It consists of lines perpendicular to the x axis, clustering near the throat. In the y direction the mesh is stretched geometrically. The same $y^+$ is used on both the top and bottom walls, with the same geometric-progression ratio. At least 1-2 grid points near the wall are in the range of $y^+ < 1$. Gerolymos and Vallet (1996), among others, suggested that the distance of the first grid point to the wall is the most important parameter concerning the grid quality in the near-wall turbulent modeling and that this value can be selected as one of the parameters to test grid independence. Three sets of grids are investigated herein: a $161 \times 65$ grid with $y^+=0.7$, a $321 \times 65$ grid with $y^+=0.8$, and a $321 \times 65$ grid with $y^+=0.85$. The computed stream-wise pressure distributions along the bottom wall for the steady strong-shock case are shown in Fig. 3. Comparison of the three results shows no significant differences between the solution on the $321 \times 65$ grid with $y^+=0.8$ and that on the $321 \times 65$ grid with $y^+=0.7$. The computational results presented in the following are therefore conducted mostly on the grid $321 \times 65$ with $y^+=0.8$.

The convergence history for the steady strong-shock case using a $321 \times 65$ mesh with $y^+=0.8$ is shown in Fig. 4 for the residuals of the $k$, $\omega$, and mass equations and the lag equation. Within 500 iterations, the residuals of the $k$, $\omega$ and $\mu_t$ equations are reduced by one order of magnitude while the residual of the mass equation is reduced by 3-4 orders of magnitude. It is reported from NPARC validation of transonic diffuser, special computation strategy is used to obtain convergence when using the Chien $k$-$\varepsilon$ model. The calculation is started from a uniform flow, running 5000 iterations in laminar mode with a low downstream pressure. Then, the proper downstream pressure is set for the weak-shock case and the computation is run for 1000 iterations using the SST model. After that, the Chien $k$-$\varepsilon$ model is initialized and run for 10000 iterations before finally the proper downstream pressure for the strong-shock case is set and the code is run for 10000 iterations. In contrast, the calculation here is initialized with a uniform condition and the back-pressure is imposed directly for the strong-shock value. No special strategy is used for convergence. Although no convergence history data from the NPARC studies is available for our comparison, the computational results presented later show that the results obtained within 300 iterations with 4 levels of multigrid is sufficiently converged and accurate for comparison with the experimental data. Undoubtedly a finer resolution that lowers the aspect ratio of the grid helps.

**Steady Case**

Figs. 5(a)-(b) show the pressure distributions on the top and bottom walls for the steady weak-shock case. For comparison, the experimental results of Sabjen et al. (1984) are also included. No significant differences are observed between the results obtained with and without the lag model. This fact implies that the lag model has no effect on the pressure distribution for the weak-shock case. The results of the strong-shock case are shown in Figs. 6(a)-(b). The results with the lag model are closer to the experimental data than the corresponding results without the lag model.

The velocity profiles at different stream-wise locations for the weak-shock and strong-shock cases are shown in Figs. 7(a)-(d) and Figs. 8(a)-(d), respectively, along with the experimental data from Sabjen et al. (1984). The computational results agree reasonably well with the experimental data for the weak-shock case. There is also no noticeable difference between the results with and without the lag model. The computational results
for the strong-shock case do not show as good agreement with the experimental data as those for the weak-shock case. The computation with the lag model, however, gives significant improvement over that without the lag-model.

The major difference between the weak-shock and the strong-shock cases is the existence of a shock-induced separation in the strong-shock case. Figs. 9(a) and (b) show the computed (with the lag model) velocity vectors under the weak-shock and the strong-shock conditions. It is clearly seen that there is a separation bubble on the top wall, which extends to about x/hth=6, for the strong-shock case. No separation is observed for the weak-shock case.

It is well known that flow history information in a turbulence model significantly influences the accuracy of a computation, especially for separated flows. This history effect is partially taken into account in a general two-equation turbulent model like the k-ω model. However, it is found that turbulence adjusts to equilibrium on a time scale much slower than that for the change of the mean strain-tensor estimated by the normal two-equation models. The principal effect of the lag model is to reduce the Reynolds stress from the equilibrium value \( V_\mu \) to a non-equilibrium lagged value \( V'_\mu \). This is important for flows with separation, which are out of “equilibrium”.

The effect of the lag model on the prediction of flow separation is shown in Figs. 10(a)-(b) for the strong-shock case. The results obtained by the lag model show a slightly larger flow separation region than the corresponding results without the lag model. Comparisons of the computed separation and reattachment points and separation lengths with the experimental data and two computational results of NPARC are listed in Table 2. The two NPARC results listed here are from the NPARC validation archive for the transonic diffuser. They are WIND k-ε without correction and WIND k-ε with two correction factors. The first correction factor is the Sarkar compressibility correction, which provides for an increase in the dissipation rate at higher Mach numbers. Another correction factor is the variable \( C_\mu \) option, which reduces the turbulent viscosity in regions where the ratio of production to dissipation of turbulent kinetic energy becomes large.

The comparison reveals that the separation length obtained with the lag model is closer to the experimental data than the corresponding results without the lag model. However, it seems that the locations of separation and reattachment predicted by the lag model shift slightly downstream compared to the experimental results. The lag model also predicts a more accurate separation length than the WIND k-ε model with or without the two correction factors. The separation and reattachment locations obtained by the present lag model are better than those obtained by the WIND k-ε without the correction factors, but worse than those with the two correction factors.

The WIND SST model results are also included in Table 2. It shows good agreement with the separation location, but the separation length is over-predicted. The present result with the lag model proves to be the best among the results listed in Table 2. This also indicates that results may be further improved by using the SST model coupled with the lag model. However, the SST model requires the use of a length scale which complicates the computation for complex configurations.

### Table 2 Comparison of flow separation (strong-shock)

<table>
<thead>
<tr>
<th></th>
<th>Separation location (x/hth)</th>
<th>Reattachment location (x/hth)</th>
<th>Separation length (( \Delta x/hth ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>1.98</td>
<td>6.00</td>
<td>4.02</td>
</tr>
<tr>
<td>WIND k-ε (no correction)</td>
<td>2.5</td>
<td>4.59</td>
<td>2.09</td>
</tr>
<tr>
<td>WIND k-ε (Sarkar+Var.( C_\mu ))</td>
<td>2.2</td>
<td>5.96</td>
<td>3.76</td>
</tr>
<tr>
<td>WIND SST</td>
<td>2.0</td>
<td>6.79</td>
<td>4.79</td>
</tr>
<tr>
<td>Normal k-ω</td>
<td>2.65</td>
<td>5.93</td>
<td>3.28</td>
</tr>
<tr>
<td>k-ω plus Lag model</td>
<td>2.29</td>
<td>6.32</td>
<td>4.03</td>
</tr>
</tbody>
</table>

### Unsteady Case

The calculation of unsteady flow starts from a steady solution, the final flow-field becomes quasi-periodic depending on the pressure ratio \( R_p \). For the weak-shock case, the flow becomes periodic after 2-3 periods, while for the strong-shock case, 5-6 cycles are needed to achieve a fully developed periodic flow. Within each implicit time step, 30-40 multigrid cycles are used. Four levels of multigrid are used for accelerating the convergence.

### Weak-Shock

Figs. 11(a)-(b) show the computed stream-wise distributions of the time-mean velocity and pressure in the midstream of the diffuser for the weak-shock case. The velocity is non-dimensionalised by the sound speed at the throat (\( a_0 \)). The experimental results are also included for comparison. The computational results are in good agreement with the experimental data. In addition, no apparent differences between the results with and without the lag model can be observed from these figures. This fact implies that, for this unsteady weak-shock case, the effect of the lag model on the time-mean values is not significant. This resembles the conclusion found in the steady weak-shock case.

The variations of the unsteady total pressure and the static pressure at midstream and the static pressure on the top wall, all at x/hth=5.836, are shown in Figs.
12(a)-(b) within one oscillation cycle. The results show that all pressure oscillations nearly follow a sinusoidal form, similar to that of the imposed backpressure. Moreover, the differences between the results with and without the lag model are negligible. Figs. 13(a)-(b) show that the amplitude and phase angle distributions along the stream-wise direction for the midstream pressure. The amplitude is normalized here by the amplitude of the back-pressure. The computational results are generally in agreement with the experimental results. No apparent effect of the lag model can be observed for this weak-shock case. The movement of shock location at different time instants within one period is shown in Fig. 14. The shock moves upstream and downstream with the back-pressure fluctuation. It can be concluded from the above results that the lag model has almost no effect on the computational results for the weak-shock case, in which there is no flow separation.

Strong-Shock
The time-mean stream-wise velocity and pressure distributions for the case with $R_p=0.72$ are shown in Figs. 15(a)-(b). The experimental results are also included for comparison. In contrast to the weak-shock case, the computational results by the lag model are closer to the experimental data for this case. This fact demonstrates that the lag model, which is effective for predicting separated flow, has an improvement on the calculation.

Figs. 16(a)-(c) show the variations of total and static pressures at midstream and the pressure on the top wall at $x/h=5.836$ within one oscillation cycle. In contrast to the results of the weak-shock case shown in Figs. 12(a)-(b), the differences in pressure with and without the lag model can be clearly seen from these figures. The streamwise distributions of amplitude and phase angle of the midstream pressure for the strong-shock case are shown in Figs. 17(a)-(b) along with the experimental data. Both figures show that slight improvements are achieved with the implementation of the lag model. Compared to the weak-shock case, the amplitude and phase angle of the time-dependent midstream pressure are no longer monotonically decreasing with the stream-wise distance ($x/h$) as the weak-shock results do. Here, they initially increase and then decrease with $x/h$ after reaching their maximum values.

4. CONCLUSIONS
The lag model in addition to the $k$-$\omega$ turbulent model is applied to simulate the 2-d steady and unsteady convergent/divergent transonic diffuser flows using a finite-volume method. For the unsteady flow, the dual-time approach is used to discretise the unsteady equations with an implicit time stepping method for the solution of the steady-state problem in pseudo-time. Results are presented for two cases with a weak and a strong shock, respectively. The main conclusions can be summarized as follows:

1) Improved performance is obtained by using the lag model for the strong-shock case in which there is flow separation. The predictions with the lag model are in closer agreement with experimental results. The predicted separation length is remarkably more accurate than that without the lag modification.

2) For the weak-shock case, no significant differences between the results with and without the lag model are observed.

3) The above conclusions are drawn for both the steady and unsteady cases investigated. The lag model is a simple addition to model the non-equilibrium effects for the eddy viscosity in both steady and unsteady cases.

REFERENCES
Fig. 1 Control volume.

Fig. 2 Grid distribution (321x65 mesh, $y^+ = 0.8$)

Fig. 3 Pressure distributions on the bottom wall computed on different grids for the strong-shock case.

Fig. 4 Convergence history (321x64 mesh, $y^+ = 0.8$)

Fig. 5 Pressure distributions along top and bottom walls, weak-shock case. (a) top wall; (b) bottom wall. Solid diamond: experiment; dashed line: with the lag model; solid line: without the lag model.
Fig. 6 Pressure distributions along top and bottom walls, strong-shock case. (a) top wall; (b) bottom wall. Solid diamond: experimental results; dashed line: results without the lag model; solid line: results with lag model.

Fig. 7 Velocity profiles at different streamwise locations, weak-shock case. (a) x/hth = 1.73; (b) x/hth = 2.88; (c) x/hth = 4.61; (d) x/hth = 6.34. Solid diamond: experiment; dashed line: without the lag model; solid line: with the lag model.
Fig. 8  Velocity profiles at different streamwise locations, strong–shock case.  (a) x/hth = 2.88;  (b) x/hth = 4.61;  (c) x/hth = 6.34; (d) x/hth = 7.49. Solid diamond: experiment; dashed line: without the lag model; solid line: with the lag model.

Fig. 9  Velocity vectors. (For clarity, the vectors are drawn by skipping 4 grid lines in the stream-wise direction and 2 grid lines in the y direction.) (a) weak-shock case (b) strong -shock case.
Fig. 10 Velocity vector in the separation region for the strong-shock case. (For clarity, the vectors are drawn by skipping 4 grid lines in the streamwise direction and 2 grid lines in the y direction.) (a) with the lag model; (b) without the lag model.

Fig. 11 Distributions of midstream time-mean velocity and pressure, weak-shock case. (a) velocity (b) pressure. Solid diamond: experiment; dashed line: without the lag model; solid line: with the lag model.

Fig. 12 Variations of midstream total pressure and static pressure and top wall pressure within one period at x/hth=5.836. (a) midstream total pressure. (b) midstream static pressure and top wall pressure. Solid lines: with the lag model; Dashed lines: without the lag model.
Fig. 13  Distributions of amplitude and phase angle of midstream pressure, weak-shock case. Solid diamond: experiment; Open circle: with the lag model; Open triangle: without the lag model.

Fig. 14  Movement of shock location at different time instants within one period.
Fig. 15  Distributions of midstream time-mean velocity and pressure, strong –shock case. (a) velocity; (b) pressure. Solid diamond: experiment; dashed line: without the lag model; solid line: with the lag model.

Fig. 16  Variations of midstream static and total pressure and top wall pressure within one period at x/hth=5.836, strong –shock case. (a) midstream total pressure; (b) midstream static pressure; (c) top wall pressure. Solid lines: with the lag model; Dashed lines: without the lag model.

Fig. 17  Distributions of amplitude and phase angle of midstream pressure, strong –shock case. Solid diamond: experiment; Open circle: with the lag model; Open triangle: without the lag model.