Investigation of Mistuning Effects on Cascade Flutter Using a Coupled Method

Mani Sadeghi and Feng Liu
Department of Mechanical and Aerospace Engineering
University of California, Irvine, CA 92697-3975
Investigation of Mistuning Effects on Cascade Flutter Using a Coupled Method

Mani Sadeghi* and Feng Liu†
Department of Mechanical and Aerospace Engineering
University of California, Irvine, CA 92697-3975

Effects of frequency mistuning on cascade flutter are studied using a method with coupled structural dynamics and aerodynamics. The unsteady structural and Euler equations are simultaneously integrated in time. An implicit finite-volume scheme of second-order accuracy is used to solve the flow equations. The structural equations for a rigid blade profile are solved with a modal approach. Investigations are performed on a turbine cascade with flutter in bending mode and with alternate mistuning of the structural eigenfrequency. It is shown that fluid-structure interaction tends to decrease the effective amount of mistuning. There exists a minimum amount of mistuning required to stabilize the cascade. The same qualitative behavior is shown to exist with a compressor cascade.

I. Introduction

In flutter investigations a turbomachinery cascade is usually assumed to be tuned, which means that all blades in the row have identical structural and geometrical properties. However, due to manufacture tolerances a real blade row exhibits variations in blade mass, geometry and stiffness. A cascade in which not all blades are identical is said to be mistuned.

In the past, several studies have been performed in order to find out if mistuning has a negative effect on flutter stability or forced response. In many cases it was found that mistuning does not increase the instability. On the contrary, it may even be beneficial. Generally, mistuning seems to increase the flutter stability, whereas the qualitative effect on forced response depends mainly on the forced mode of vibration. Therefore, in case of flutter, it was suggested to intentionally mistune a cascade in certain patterns. In particular, alternate frequency mistuning, where the structural eigenfrequencies of neighboring blades are alternating between a high and a low value, was found to have a stabilizing effect.

Kaza and Kielb performed a modal analysis of tuned and mistuned cascades of pre-twisted fan blades. Linear theory for a flat plate was applied to solve for the flow ranging from subsonic to supersonic along the rotor span. An unshrouded fan stage, unstable at the design point, was shown to be stabilized by alternate mistuning, with a frequency variation of 7%.

Inregun and Ewins performed numerical studies on a cascade of flat plates, in the incompressible, subsonic and supersonic Mach number range. The structural behavior was modeled with a lumped parameter presentation of rigid blade profiles, allowing for structural coupling between the blades. Alternate mistuning was found to stabilize critical vibration modes at the expense of damped ones.

An experimental investigation on forced response of naturally and artificially mistuned propfan blades was done by Mehta and Murthy. Artificial mistuning was performed by varying the ply orientation of the composite material of the blades, causing a variation of stiffness and vibration mode shape. This intentional mistuning was found to cause a large reduction in the forced response of higher responding blades at relatively little change in the amplitudes of lower responding blades, and therefore, alternate mistuning had an overall stabilizing effect.

An experimental and numerical investigation on an annular turbine cascade was done by Nowinski and Panovsky. The blades were oscillated in a harmonic torsional mode. Three vibrational modes of the blades were tested: the traveling wave mode, the single blade mode, and the alternating blade mode. In the last test mode, only alternate blades in the cascade were excited in a traveling wave pattern while others remained stationary to simulate frequency mistuning. The authors found that alternate frequency mistuning reduced the dependence of the aerodynamic damping coefficient on the IBPA and significantly enhanced the stability of the tested low pressure turbine cascade.

In a previous work mistuning was studied in the time domain by directly solving the unsteady flow of a cascade under mistuned oscillations. A multigrid time-accurate Navier-Stokes code is used to calculate quasi-three-dimensional unsteady flows around multiple oscillating turbine blades. The code is made parallel by using MPI so that multiple passages can be calculated without the use of phase-shifted periodic boundary conditions.

*Graduate Researcher, AIAA Student Member
†Associate Professor, AIAA Member
Copyright ©2001 by the authors. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission.
The uncoupled approach was used in which the oscillation frequencies and amplitudes of all blades are specified and stability is determined by calculating the aerodynamic damping coefficient as defined by Böics and Fransson. The standard configuration 4 of a turbine cascade compiled by Böics and Fransson was used as a test case. Damping coefficients were obtained for various inter-blade phase angles for the tuned case and compared with results for both phase-shift mistuning and frequency-mistuning. It was found that mistuning of phase-shift has small effects on the flutter characteristics. However, mistuning of frequency has the effect of averaging out the damping coefficient for the tuned blade row over the whole range of IBPA because of a temporally changing phase difference between each blade and its neighbors.

In summary, several studies have shown numerically that frequency mistuning has the effect of increasing the flutter stability by distributing the energy of unstable flutter modes over a range of stable modes. A mistuned cascade is not able to oscillate in a mode of constant IBPA. Therefore, at any time, several modes of different IBPAs are present and as long as most of these modes are stable, the cascade will be overall stable. These findings have been confirmed by a perturbation analysis of Campobasso and Giles.

However, none of the above investigations answers the question about the minimum amount of alternate mistuning that is required for stabilization. If the above description for the stabilizing mechanism of mistuning were complete, it would work at any finite amount of mistuning. It is obvious that this cannot be true, because we know that a real blade row is slightly mistuned but may still flutter. In order to investigate the effects of mistuning more accurately, a more adequate model of the physics has to be applied by using a coupled method that accounts for the interaction between fluid and structure.

The uncoupled method implies the assumption that the blade mass ratio is sufficiently high so that blades are oscillating at their structural eigenfrequencies and at constant amplitude. It was suggested that the definition of stability, based on the damping coefficient as defined in Ref. 6 may not be suitable in case of coupling between aerodynamics and structural dynamics. With frequency mistuning, the uncoupled method predicts a temporal maximum in the work done on each blade. This maximum exists even if the overall behavior is stable, because due to a time dependent IBPA, the cascade will temporally go through an IBPA range in which the oscillation is unstable. If coupling between aerodynamics and structural dynamics is present, the work done on a blade will affect the blade’s oscillation amplitude. Therefore, even in situations where the energy method predicts stability the amplitude may temporally exceed a critical level. To prove this conjecture, flutter investigations have to be performed with a coupled approach.

In order to perform coupled computations, the present authors included a structural model to account for fluid-structure interactions. An integrated method is used, which solves the Navier-Stokes and structural equations simultaneously in each real-time step. The elastic behavior of a blade is modeled with a linear spring for the bending motion and a torsional spring for the rotational degree of freedom. The method was used to investigate non-linear flutter, comparing results by the Navier-Stokes equations with solutions by the Euler equations in Ref. 8.

In this work the effects of fluid-structure coupling on frequency mistuning are investigated. It is shown that the interaction between structural dynamics and aerodynamics has a destabilizing effect on a mistuned cascade. Due to fluid-structure interaction the actual oscillation frequency differs from the mistuned structural eigenfrequency. The effective mistuning is not given by the variation of the structural eigenfrequency but by the variation of the actual oscillation frequency. It appears that fluid-structure coupling tends to diminish the differences between the oscillation frequencies of neighboring blades.

II. Flow Solver

For a two-dimensional control Volume V with moving boundary ∂V the unsteady quasi-three-dimensional Favre-averaged Navier-Stokes equations with a k-ω turbulence model can be written as follows:

$$\frac{\partial}{\partial t} \int_V \theta(x) \, w \, dV + \oint_{\partial V} \theta(x) \left( \mathbf{f} \, dS_x + \mathbf{g} \, dS_y \right)$$

$$= \oint_{\partial V} \theta(x) \left( \mathbf{f}_n \, dS_x + \mathbf{g}_n \, dS_y \right) + \int_V \mathbf{S} \, dV \tag{1}$$

where the vector w contains the conservative flow variables plus the turbulent kinetic energy k and the specific dissipation rate ω, in the k-ω turbulence model by Wilcox. The vectors f, f_n, g, g_n are the Euler fluxes and viscous fluxes in the x- and y-directions, respectively. This formulation is quasi-three-dimensional in the sense that it accounts for a streamwise variation of the blade span by including the streamtube thickness θ(x). A detailed description of these terms can be found in Ref. 10.

The JST scheme is used for flux discretization with an implicit formulation, using a second-order difference for the time derivative. The equations are integrated in time by Jameson’s pseudo-time stepping. The computation is performed in parallel by several processors, where each processor calculates the flow through one passage. The exchange of bound-
ary conditions between neighboring blade passages is performed using the Message Passing Interface (MPI).

III. Structural Model

The structural behavior of a rigid blade profile with two degrees of freedom is governed by two coupled ODE’s:

$$\ddot{h} + S_a \dot{\alpha} + K_b h = -L$$
$$\ddot{\alpha} + I_a \dot{\alpha} + K_a \alpha = -M_e \alpha$$  \hspace{1cm} (2)

where $h$ and $\alpha$ are the translational and rotational displacements, $L$ and $M_e$ are the aerodynamic force component in the direction of the displacement and the aerodynamic moment about the elastic axis; $K_b$ and $K_a$ are the bending and torsional spring stiffnesses; $S_a = m b x_o$ is the static unbalance of the profile, which provides the coupling between the plunging and pitching degrees of freedom; $I_a = m b^2 r_o^2$ is the area moment of inertia of the profile about the elastic axis; and $m$ is the mass per unit span. The translational displacement $h$ is non-dimensionalized with the semicord $b$. The pitching and plunging stiffnesses are modeled as stiffnesses of linear springs attached at the elastic axis. Time is non-dimensionalized with the eigenfrequency of the pitching mode, i.e. $\tau = \omega_b t$.

In terms of these dimensionless quantities, the equations of motion for the aerodynamic systems are:

$$\frac{d^2}{d\tau^2} \left( \frac{h}{b} \right) + x_o \frac{d^2}{d\tau^2} \left( \frac{\alpha}{\omega^2 b} \right) + \frac{\omega^2}{\omega^2 b} \frac{h}{b} = -\frac{C_l}{\pi \mu k^2 c (\omega/\omega_b)^2}$$
$$x_o \frac{d^2}{d\tau^2} \left( \frac{h}{b} \right) + \frac{r_o^2}{\omega^2 b} \frac{d^2}{d\tau^2} \left( \frac{\alpha}{\omega^2 b} \right) + r_o^2 \frac{\alpha}{\omega^2 b} = -\frac{2C_m}{\pi \mu k^2 c (\omega/\omega_b)^2}$$  \hspace{1cm} (3)

where $C_l$ and $C_m$ are the lift and moment coefficient about the elastic axis, and $k_c = \omega_c c / 2U_\infty$ is the reduced frequency. We rewrite the above equations in the more familiar form

$$[M] \ddot{q} + [K] \dot{q} = \mathbf{F}$$  \hspace{1cm} (4)

where

$$[M] = \begin{bmatrix} 1 & \frac{x_o}{r_o^2} \\ \frac{x_o}{r_o^2} & 1 \end{bmatrix}$$
$$[K] = \begin{bmatrix} \frac{(\omega/\omega_b)^2}{\omega^2 b} & 0 \\ 0 & \frac{r_o^2}{\omega^2 b} \end{bmatrix}$$

are the non-dimensional mass and stiffness matrices, and

$$\mathbf{F} = \frac{1}{\pi \mu k^2 c (\omega/\omega_b)^2} \begin{bmatrix} -C_l \\ 2C_m \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} \frac{h}{b} \\ \frac{\alpha}{\omega^2 b} \end{bmatrix}$$

are the load and displacement vectors.

The modal structural equations are discretized and cast into a pseudo-time problem similar to the treatment of the flow equations. A Runge-Kutta scheme is used to simultaneously advance the structural and flow equations in pseudo-time until convergence within each real-time step. Notice that the aeroelastic equations are implicitly coupled to the flow equations since the forcing term contains both $C_l$ and $C_m$. By simultaneously iterating the flow and structural equations with the same pseudo-time approach, this coupling is taken into account. Details of the method can be found in Ref. 8.

IV. Results

A. Uncoupled Method

With the uncoupled approach, the cascade behavior is periodic because of the specified sinusoidal blade motion. In that case, the work that the aerodynamic forces perform on a blade over one oscillation cycle is used as a measure for stability. If this work coefficient is positive, the cascade is unstable at the given IBPA.

Figure 1 shows the work coefficient as a function of the inter-blade phase angle for standard configuration 4 by Böls and Fransson, which is a cascade of turbine blades oscillating in pure plunging mode in high subsonic flow. For pure bending the work coefficient is defined as

$$C_W = \frac{T_{tuned}}{T_{total}} \int_0^{T_{tuned}} C_B \, dh$$  \hspace{1cm} (5)

where $h$ is the translational displacement divided by the chord length, $C_W$ is the work coefficient, $C_B$ the force coefficient in $h$ direction, $T_{tuned}$ the period of the tuned case, and $T_{total}$ a long overall period of the system determined by the amount of mistuning.

For the purpose of this investigation, the Euler equations are solved, i.e. the viscous and turbulent terms in Eq. (1) are disregarded. The calculation is performed on a number of blade passages, depending on the imposed inter-blade phase angle. These results were obtained by the uncoupled method. As shown in Fig. 1, the cascade exhibits flutter instability in a range of IBPAs between 240° and 360°.

However, when the cascade is subject to alternate mistuning, where the oscillation frequency of every second blade is increased over the nominal frequency by a given amount $\Delta \omega$, the cascade is stabilized. In Fig. 2 the time history of work done on the cascade is shown for mistuning levels of $\Delta \omega = 2\%, 5\%$ and 10% of the nominal frequency $\omega$. Here, time $t$ is non-dimensionalized by the nominal oscillation period $T$. These results were obtained by calculations on four blade passages allowing for the critical IBPA of 270°.

The low frequency difference $\Delta \omega$ introduced by mistuning every second blade in the row is apparent in Fig. 2. The phase difference between neighboring
Fig. 1 Work coefficient of the tuned cascade versus inter-blade phase angle, uncoupled.

blades goes through the whole range between 0° and 360° within each period 2π/Δω. One could think of each blade going through the unstable and stable regions shown in Fig. 1 as the phase difference changes with time. The slope of the curves in Fig. 2 is associated with instability where positive, and with stability where negative. For all cases shown in Fig. 2, the slope averaged over one period \( T_{\text{total}} = 2\pi/\Delta \omega \)

\[
\frac{dC_W}{dt} = \frac{1}{T_{\text{total}}} \int_0^{T_{\text{total}}} \frac{dC_W(t)}{dt} dt = \frac{C_W(T_{\text{total}})}{T_{\text{total}}}
\]

is shown with the straight solid line. In the case of a tuned cascade, this slope would correspond to the work coefficient at the given IBPA.

With alternate mistuning, and thus temporally changing phase differences, the average slope is close to the IBPA averaged work coefficient

\[
\overline{C_W}(\text{IBPA}) = \frac{1}{2\pi} \int_0^{2\pi} C_W(\text{IBPA}) d\text{IBPA}
\]

shown with the dashed line in Fig. 2. It is remarkable that within the investigated range of mistuning levels, the work performed on the cascade seems to fall on a common average slope. The difference between this average slope and the IBPA averaged work coefficient obtained from Fig. 1 is mainly due to the fact that the phase difference between blade \( n \) and blade \( n-1 \) is given by

\[
\phi_n - \phi_{n-1} = \omega t + \phi_{0,n} - (\omega + \Delta \omega)t - \phi_{0,n-1} = -\Delta \omega t + \Delta \phi_{0,n,n-1}
\]

but between blade \( n+1 \) and blade \( n \) the phase difference becomes

\[
\phi_{n+1} - \phi_n = (\omega + \Delta \omega)t + \phi_{0,n+1,n} - \omega t - \phi_{0,n} = +\Delta \omega t + \Delta \phi_{0,n+1,n}
\]

Fig. 2 Time history of work done on the cascade with alternate mistuning at various levels, initial IBPA = 270°, uncoupled.

where \( \phi_0 \) are initial phase angles. Thus, the phase differences to the suction side neighbor and to the pressure side neighbor are different at most times. The influence of the neighbor on the suction side is dominant, but the influence of the pressure side neighbor and second neighbors may explain why the average temporal derivative of the cascade work deviates from the average work coefficient.

With the uncoupled approach the work coefficient in Eq. (5) is used as the single measurement for stability. In the case of frequency mistuning, however, the work coefficient is not a suitable measure because it only provides information about the time average behavior. Figure 2 shows that even though the overall behavior is stable, the mistuned cascade exhibits a temporal maximum in work which is higher at lower levels of mistuning.

Thus, initially the cascade extracts energy from the flow until a maximum is reached and the energy decreases. If for a cascade with fluid-structure interaction this maximum corresponds to a maximum oscillation amplitude, there will be a requirement for a minimum mistuning level at which the cascade can be regarded as stable. A coupled method has to be applied in order to account for fluid-structure interaction.

B. Coupled Method

The uncoupled method is valid only if the blade mass ratio in Eq. (4)

\[
\mu = \frac{m}{\pi \rho \infty b^2}
\]

is high enough so that the structural behavior is not affected by aerodynamic forcing. Here, \( m \) is the blade mass per unit span, \( \rho \infty \) the density at the inlet, and \( b \) the half-chord of the blade profile. When investigating the effects of mistuning, the fluid-structure interaction may be significant even at high mass ratios. At a high
mass ratio the aerodynamic forcing is small compared to inertial forces. However, if the effect of mistuning is also small, the coupling effect may not be negligible.

All following results were obtained by the coupled approach, simultaneously solving for the flow and the structure dynamics on four blade passages.

**B.1 Turbine Cascade**

In Fig. 3 the total cascade energy is shown over time for the tuned standard configuration 4 and an IBPA of 270°. The total energy of the cascade is given by the sum of the kinetic and potential energies of all blades

\[ E = \frac{1}{2} \sum_m \left( \mathbf{q}_m \cdot \dot{\mathbf{q}}_m + \frac{\omega^2}{2} \mathbf{q}_m \cdot \mathbf{q}_m \right) \]

where \( m \) is the blade index. The tuned cascade is unstable as was predicted by the uncoupled method. Using the coupled approach we find that the instability increases as the mass ratio is decreased. The cascade energy increases more rapidly at lower mass ratios in Fig. 3, which implies that the fluid-structure interaction has a destabilizing effect.

**Fig. 3** Total energy of the tuned cascade versus time.

**Fig. 4** Time history of the cascade energy for mass ratio 500.

**Fig. 5** Total energy of the cascade versus time for a mass ratio of 200.

**Fig. 6** Total energy of the cascade versus time for a mass ratio of 500.

Figure 4 shows the time history of the total energy of the cascade for a mass ratio of 500 and alternate frequency mistuning by 1%, 2%, 3%, 5% and 10%. For mistuning amounts above 2% the cascade is stable. However, there is a maximum total energy, i.e. a maximum deflection, which increases as \( \Delta \omega \) is decreased. This confirms the conjecture earlier drawn from uncoupled results by the authors.\(^5\)

However, Fig. 4 also shows that mistuning by only 1% does not stabilize the cascade. The reason for this surprising result is given by the influence of fluid-structure coupling on the blade oscillation frequency. It should be noted that due to fluid-structure interaction a blade is not oscillating at its structural eigenfrequency. The actual oscillation frequency differs from the eigenfrequency, and this difference increases as the mass ratio is decreased. With this test case, the influence of fluid-structure coupling apparently decreases the effective mistuning. Since this phenomenon is due to fluid-structure interaction, it cannot be predicted by the uncoupled approach.
Fig. 7 Total energy of the cascade versus time for a mass ratio of 800.

More detailed plots of the time history of the cascade energy are given in Figs. 5 through 7 for the mass ratios of 200, 500 and 800. Note that the energy is plotted on a logarithmic scale, where a linear behavior indicates exponential decay or growth. For each mass ratio, the energy is plotted for several mistuning amounts, and an attempt was made to find the minimum amount necessary for stabilization. We can see from Figs. 5 through 7 that this minimum decreases with increasing mass ratio. This is because the coupling between fluid and structure decreases as the mass ratio is increased. In the limit of an infinite mass ratio, the aerodynamic equations and the structural equations are decoupled and we would expect to find the result of the uncoupled method, i.e. stabilization at all mistuning amounts.

A temporal maximum of the cascade energy is present at all stable solutions and is plotted over the amount of mistuning in Fig. 8 for the three mass ratios $\mu$. Again, this figure shows that the cascade is unstable below a certain finite amount of mistuning, which becomes smaller as the mass ratio is increased. Stable solutions exhibit a maximum energy which increases and approaches infinity as the mistuning amount is decreased and approaches the stability limit. Rather than using the damping coefficient or the mistuning amount, it may be more appropriate to define the stability limit as a horizontal line in Fig. 8 to mark a maximum allowable cascade energy. In order to yield accurate quantitative solutions, however, a three-dimensional configuration should be investigated solving the Navier-Stokes equations coupled to the structural equations of three-dimensional blades.

In a real cascade, the actual oscillation frequencies are not known a priori and are hard to control in advance. Structural eigenfrequencies can be controlled more easily by controlling the blade stiffness through the choice of blade material and structure. For this reason, by mistuning we mean mistuning in structural eigenfrequency, not in actual oscillation frequency. Because of fluid-structure interaction, the actual oscillation frequency is not equal to the structural eigenfrequency.

Figure 9 helps to explain the existence of the lower limit for the mistuning amount. The effective mistuning, i.e. the difference present in the actual oscillation frequencies of adjacent blades, is plotted over the nominal mistuning, i.e. the applied difference of structural eigenfrequencies. The results for mass ratios of 200, 500 and 800 are shown together with the uncoupled solution ($\mu = \infty$). For the uncoupled case, where the blades are indeed oscillating at their eigenfrequencies, the applied amount of mistuning is identical to the mistuning of the actual oscillation frequency.

Note that for finite mass ratios, below a certain structural mistuning amount, the cascade stays essentially tuned and is therefore unstable. Interestingly, as the structural mistuning is increased above this limit, the actual mistuning resumes to increase at the originally expected rate. In this plot, the influence of fluid-structure coupling appears to add a free-play of mistuning to the system.

B.2 Compressor Cascade
Fig. 10 Total energy of the tuned cascade versus time.

As a second test case the tenth standard configuration by Fransson and Verdon was chosen, which is a cascade of modified NACA 0006 profiles. At an inlet Mach number of 0.5, an inlet angle of 48° and torsional motion around midchord, this cascade exhibits flutter at inter-blade phase angles around 90°. The following results were obtained by calculations on four blade passages with the coupled approach.

As with the turbine cascade presented in the previous section, the stability of the tuned compressor cascade decreases as the mass ratio is decreased in Fig. 10. The total energy of the tuned cascade increases more rapidly at stronger fluid-structure coupling.

Figures 11 and 12 show the time histories of the cascade energy for the two mass ratios of 180 and 350 and for various amounts of alternate mistuning. Relatively large amounts of mistuning are necessary to stabilize this compressor cascade, compared to the turbine cascade. Since compressor blades have a lower mass ratio than solid turbine blades, the effects of fluid-structure interaction are stronger. Coupling has a diminishing effect on the effective mistuning. Therefore, the stronger the coupling is, the more mistuning in eigenfrequency is needed to overcome this effect. Also the compressor cascade exhibits a maximum cascade energy which decreases as the amount of mistuning is increased in Fig. 13.

Qualitatively, the behavior of the compressor cascade is very similar to the behavior of the turbine test case. Due to the lower mass ratio of the compressor blades, however, there is stronger fluid-structure interaction than with the turbine cascade. Therefore, the minimum amount of mistuning required to stabilize the compressor cascade is higher than that for the turbine cascade.

Fig. 11 Total energy of the cascade versus time for a mass ratio of 180.

Fig. 12 Total energy of the cascade versus time for a mass ratio of 350.

Fig. 13 Maximum cascade energy over mistuning amount.
V. Conclusions

Frequency mistuning may stabilize a cascade by causing temporally changing phase differences between adjacent blades. These temporal changes have an averaging effect on the stability of unstable and stable oscillation modes. With alternate mistuning, the resulting work coefficient, however, is not exactly given by the IBPA-averaged work coefficient of the tuned cascade. This discrepancy is mainly due to different contributions of the suction side neighbor and the pressure side neighbor of each blade.

Results by the coupled method and alternate frequency mistuning confirm the existence of a maximum total energy associated with a temporal maximum in blade deflection even at stable situations. Thus, depending on the mass ratio, a minimum mistuning level $\Delta \omega$ is required, corresponding to a maximum allowable deflection amplitude. This maximum amplitude decreases as the amount of mistuning is increased.

Furthermore, with the presented turbine and compressor test cases fluid-structure coupling has a diminishing effect on the mistuning level. The effective amount of mistuning is given by the difference in the actual oscillation frequencies of neighboring blades and not by the difference of the structural eigenfrequencies. If fluid-structure coupling is present, the actual oscillation frequency may differ from the structural eigenfrequency of a blade. By this mechanism a structurally mistuned cascade is able to oscillate in a quasi tuned fashion. Since the tuned oscillation is less stable than mistuned oscillation, the blades in a cascade will naturally tend to oscillate at the same frequency. The ability to overcome the structural differences increases as the mass ratio is decreased, because of an increase in fluid-structure coupling.

Thus, the mistuning in eigenfrequency must exceed a certain limit in order to be effective. The stronger the fluid-structure coupling is, the larger is this minimum amount of mistuning. When the amount of mistuning is raised above the minimum and further increased, the effective mistuning also increases.

References


