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Computation of Mistuning Effects on Cascade Flutter

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Computation of Mistuning Effects on Cascade Flutter

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Abstract

This paper describes a computational method for predicting flutter of turbomachinery cascades with mistuned blades. The method solves the unsteady Euler/Navier-Stokes equations for multiple blade passages on a parallel computer using the Message Passing Interface (MPI). A second order implicit scheme with dual time-stepping and multigrid is used. Each individual blade is capable of moving with its own independent frequency and phase angle, thus modeling a cascade with mistuned blades. Flutter predictions are performed through the energy method. Both phase angle and frequency-mistuning are studied. It is found that phase-angle mistuning has little effect on stability whilst frequency-mistuning significantly changes the aerodynamic damping. The important effect of frequency-mistuning is to average out the aerodynamic damping of the tuned blade row over the whole range of Inter-Blade Phase Angle (IBPA). If a tuned blade row is stable over a majority of the IBPA range, the blades can be stabilized for the complete IBPA range through appropriate frequency-mistuning.

1 Introduction

Turbomachinery designers are striving for increased loading and reduced size and weight of compressor and turbine blade rows, particularly for aircraft engines. As such, the flutter of the turbomachinery blades may become a limiting factor in the design and performance of gas turbine engines. Accurate theoretical and computational methods in predicting the flutter boundary will enable us to achieve high performance and low cost by allowing adequate but not excessive design margins.

Flutter calculations for turbomachinery blade rows often employ Lane's traveling wave model [1], in which the adjacent blades in a blade row are assumed to vibrate at the same frequency but with a constant phase difference, the Inter-Blade-Phase-Angle (IBPA). With this model, aerodynamic responses of the blade row can be determined by using a single blade passage in order to minimize the computational effort. Consequently, a *phase-shifted* periodic boundary condition has to be applied when the IBPA is not zero. The use of the phase-shifted boundary condition and its implementation in a flow code, using the traditional "direct store" method by Erdos and Alzner [2], implies that the solution is both temporally and spatially periodic. In an actual machine, blades are never exactly identical due to manufacturing imperfections, which result in non-identical vibration frequencies and phase shifts (mistuning) of the blades in a blade row. There is also evidence to show that certain intentional mistuning may improve the flutter characteristics of a blade row.

Kaza and Kielb [3] studied effects of mistuning for a cascade oscillating in a coupled bending-torsion or uncoupled torsion mode. Aerodynamic loads were calculated based on a linearized incompressible flow method for flat plates. They found that mistuning, alternate mistuning as well as random mistuning, has a strong beneficial effect in the case of self-excited vibration. The flutter speed was increased by increasing the mistuning level. Crawley and Hall [4] developed an inverse design procedure for the optimum mistuning of a high bypass ratio fan. A linearized supersonic aerodynamic theory is used to compute the unsteady forces in the influence coefficient form. Imregun and Ewins [5] performed numerical studies on a cascade of flat plates, in the incompressible, subsonic and supersonic Mach number range. The structural behavior was modeled with a lumped parameter presentation of rigid blade profiles, allowing for structural coupling between the blades. Mistuning, alternate mistuning in particular, was found to have positive effects by stabilizing critical vibration modes at the expense of damped ones. A recent experimental and numerical investigation on an annular turbine cascade was done by Nowinski and Panovsky [6]. The blades were oscillated in a harmonic torsional mode. Three vibrational modes of the blades were tested: the traveling wave mode, the single blade

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mode, and the alternating blade mode. In the last test mode, only alternate blades in the cascade were excited in a traveling wave pattern while others remained stationary to simulate frequency mistuning. Nowinski and Panovsky found that alternate frequency mistuning reduced the dependence of the aerodynamic damping coefficient on the IBPA and significantly enhanced the stability of the tested low pressure turbine cascade.

Except reference [6], all of the above authors performed the analysis in the frequency domain by solving an eigenvalue problem of the structural flutter equations with areodynamic loads as input in the form of influence coefficients. In this paper we propose to study mistuning in the time domain by directly solving the unsteady flow of a cascade under mistuned oscillations. Stability is determined by calculating the aerodynamic damping coefficient as defined in Bölcs and Fransson [7]. While the method is limited to high mass ratio blades it provides useful insights into the flutter mechanism in view of the aerodynamics of the flow through a cascade that undergoes mistuned blade motions. In order to perform such studies, the flow is no longer considered to be periodic in either space or time. Therefore, the traditional method with phase-shifted boundary conditions cannot be used. Computation for such flows must be done over multiple passages.

In a previous work by Ji and Liu [8] a multigrid time-accurate Navier-Stokes code with a two-equation k-ω turbulence model was developed to calculate quasi-three-dimensional unsteady flows around multiple oscillating turbine blades. The code was made parallel by using MPI so that multiple passages could be calculated without the use of phase-shifted periodic boundary conditions for blade flutter problems. The code ran efficiently on regular parallel computers or networked clusters of workstations or PCs. In this paper, we extend the method by Ji and Liu [8] to studies of mistuning effects. The standard configuration 4 of a turbine cascade compiled by Bölcs and Fransson [7] is used as a test case. Damping coefficients are obtained for various inter-blade phase angles for the tuned case and compared with results for both phase-shift mistuning and frequency-mistuning. It is found that for this case, mistuning of phase-shift has small effects on the flutter characteristics, whereas mistuning of frequency has a significant effect on the damping coefficients of each blade in the row. Frequency mistuning has the effect of averaging out the damping coefficient for the tuned blade row over the whole range of IBPA because of a temporaly changing phase difference between each blade and its neighbors. therefor, if a tuned blade row is stable for a majority of the IBPA range, alternate and random frequency mistuning can stabilize the blades over the complete range of IBPA. In fact, random frequency mistuning eliminates the dependence of the aerodynamic damping coefficient on the IBPA. The effect of the degree of frequency mistuning is also discussed in view of the damping coefficient as a measure for stability.

2 Computational Method

For a two-dimensional control Volume V with moving boundary ∂V the quasi-three-dimensional Favre-averaged Navier-Stokes equations with a k-ω turbulence model can be written as follows:

$$\begin{aligned}\frac{\partial}{\partial t}\iint_V \theta(x)\,\mathbf{w}\,dV + \oint_{\partial V}\theta(x)\,\mathbf{f}\,dS_x + \theta(x)\,\mathbf{g}\,dS_y \\ = \oint_{\partial V}\theta(x)\,\mathbf{f}_\mu\,dS_x + \theta(x)\,\mathbf{g}_\mu\,dS_y + \iint_V \mathbf{S}\,dV\end{aligned} \tag{1}$$

where the vector $\mathbf{w}$ contains the conservative flow variables plus the turbulent kinetic energy k and the specific dissipation rate ω, in the k-ω turbulence model by Wilcox [9]. The vectors $\mathbf{f}, \mathbf{f}_\mu, \mathbf{g}, \mathbf{g}_\mu$ are the Euler fluxes and viscous fluxes in the x- and y-directions, respectively. $\theta(x)$ is introduced to account for variations in the streamtube thickness, and the source vector $\mathbf{S}$ includes terms due to the variation of $\theta(x)$. A detailed description of these terms can be found in [8].

A finite-volume method is used for spatial discretization. Equation (1) can then be written in semidiscrete form:

$$\frac{d\mathbf{w}}{dt} + \mathbf{R}(\mathbf{w}) = 0 \tag{2}$$

where $\mathbf{R}$ is the vector of residuals, consisting of the spatially discretized flux balance of Equation (1). Time accuracy is achieved by using a second order implicit time-discretization scheme which is recast into a pseudo-time formulation, as proposed by Jameson [10]:

$$\frac{d\mathbf{w}}{dt^*} + \mathbf{R}^*(\mathbf{w}) = 0 \tag{3}$$

For each physical time step in Equation (2) the solution is sought by solving Equation (3) for a steady-state in pseudo-time t^*. The benefit of this reformulation is that convergence acceleration techniques such as local time stepping, residual smoothing and a multigrid method can be used in pseudo-time without sacrificing time accuracy. A phase-shifted periodic boundary condition is applied at the boundaries between the passages. For tuned blade rows this condition becomes:

$$\mathbf{w}_l = \mathbf{w}_u(x, t - \sigma/\omega) \tag{4}$$

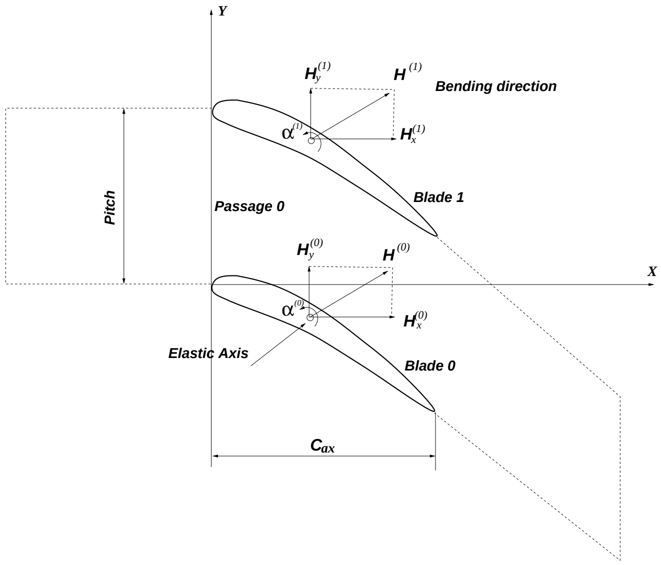

Figure 1: Blade geometry and motion definition

where subscripts l and u denote the lower and upper boundaries of a blade passage in Figure 2, σ is the IBPA, and ω is the angular frequency of the blade vibration. In order to perform calculations with mistuned blades, the conventional direct store method by Erdos and Alzner [2] used to apply the phase-shifted periodic boundary condition is replaced by pure periodic boundary conditions through the use of multiple passages as shown in Figure 2. Although this limits the inter-blade phase angle to be discrete numbers, it has the advantage of extending the calculations to a full annulus with nonperiodic motions and mistuned blades. The IBPA σ and the oscillation frequency ω can vary from blade to blade, which allows the investigation of both phase-mistuning and frequency-mistuning.

Figure 1 shows a cylindrical cut of a blade passage. The x coordinate is along the engine axis. The y coordinate is the circumferential coordinate, which is equivalent to $r\theta$ for a cylindrical cut at radius r, c is the chord length of the blade profile and c_x is the axial chord. The blades are assumed to be rigid and follow a motion of a combined bending and torsion mode. For the *m-th* blade, the bending and torsion motions can be specified as

$$\vec{h}^{(m)}(t) = a\left(\mu_x \vec{e}_x + \mu_y \vec{e}_y\right) e^{i(\omega_m t + \varphi_m)} \quad (5)$$

$$\alpha^{(m)}(t) = a(\mu_\alpha / c) e^{i(\omega_m t + \varphi_m)} \quad (6)$$

where α is positive anti-clockwise in Figure 1. If $\vec{h}(t)$ is written as

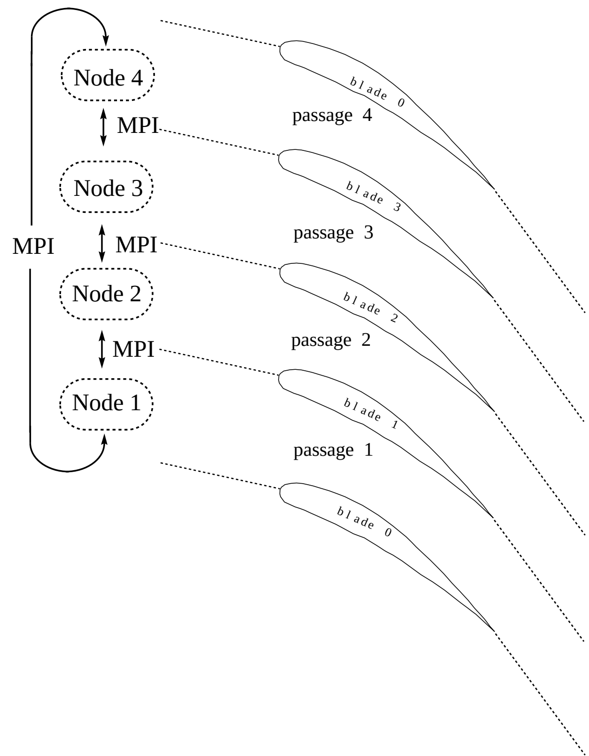

Figure 2: Multiple passage computation using MPI

$$\vec{h}^{(m)}(t) = h_x^{(m)}(t)\vec{e}_x + h_y^{(m)}(t)\vec{e}_y \quad (7)$$

we can write

$$\begin{bmatrix} h_x^{(m)}(t) \\ h_y^{(m)}(t) \\ \alpha^{(m)}(t) \end{bmatrix} = a \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_\alpha / c \end{bmatrix} e^{i(\omega_m t + \varphi_m)} \ , \ m = 0, 1, 2, ... \quad (8)$$

where m stands for the blade number; $\omega_m = 2\pi f_m$ and φ_m are the angular frequency and the phase of the forced vibration of blade m. In the tuned case, the constant inter-blade phase angle is $\sigma = \varphi_m - \varphi_{m-1}$. The parameter

$$\mu = \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_\alpha \end{bmatrix} \quad (9)$$

characterizes the specified modal shape of the combined bending and torsion motion; a is a general dimensionless amplitude. The modal parameters μ_x, μ_y, and μ_α all have the dimension of length. In general, they may be complex numbers when there are phase differences among the x, y bending motions and the torsion motion. However, standard configuration 4 does not include torsional vibration, so that the current work is performed with a pure bending mode, with a bending angle

$$\delta = \tan^{-1}(\mu_y / \mu_x)$$

A parallel algorithm is implemented in which each processor computes the flow through one blade passage, and communication between blade passages is achieved by using MPI (Figure 2). The method scales very well on both parallel computers and networked workstations. The accuracy of the numerical method and the efficiency of the parallel implementation have been validated in [8].

3 Results and Discussions

3.1 Computation for Tuned Blades

The case 552B of standard configuration 4 with inlet Mach number $M_1 = 0.28$, outlet isentropic Mach number $M_{is,2} = 0.90$ and inlet flow angle $\beta_1 = -45°$ is chosen for comparison. Earlier comparisons between Euler and Navier-Stokes results in [8] did not indicate significant viscous effects with this configuration. Therefore, in the current work only Euler calculations are performed. Figure 3 shows the steady-state isentropic Mach number distribution over the blade surface. The numerical results agree very well with the experimental data.

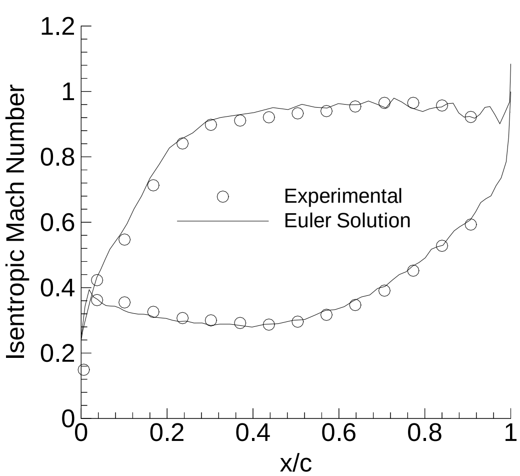

Figure 3: Steady-state isentropic Mach number distribution over the blade

The first harmonic amplitude and phase angle of the unsteady pressure coefficient are plotted in Figures 4 and 5 for the cases with $IBPA = 90°$ and $180°$. The trends of the computational and experimental data match reasonably well. However, the calculation predicts much higher amplitudes over the front half-chord, a phenomenon which was already mentioned in [7]. The

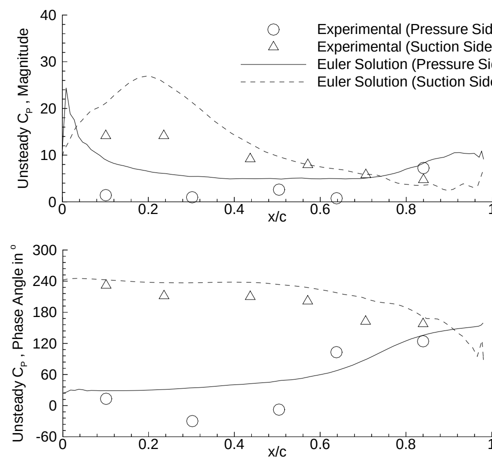

Figure 4: Amplitude and phase of the first harmonic unsteady pressure coefficient over the blade with $IBPA = 90°$

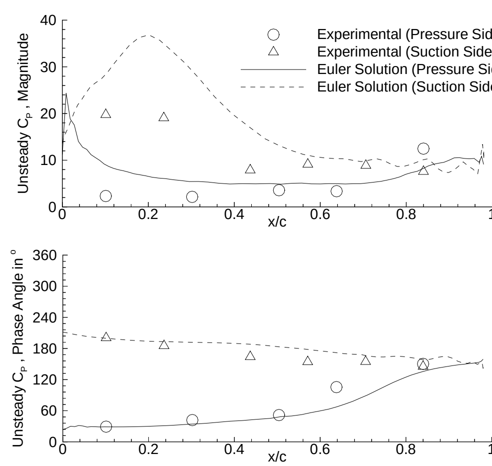

Figure 5: Amplitude and phase of the first harmonic unsteady pressure coefficient over the blade with $IBPA = 180°$

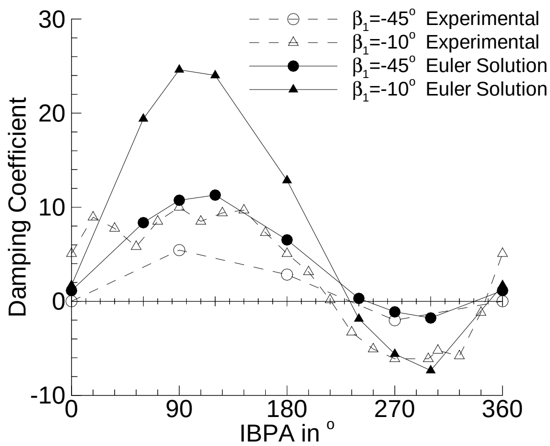

Figure 6: Damping coefficient over IBPA with $\beta_1 = -45°$ and $\beta_1 = -10°$

discrepancies in phase are small, which gives the good prediction of the stability range shown in Figure 6, even though the amplitude of the damping coefficient is large compared to the experimental data in the stable region. In the region of instability much better agreement is achieved. For the investigation of mistuning effects, the test case with $\beta_1 = -10°$ was chosen, because it is unstable over a slightly larger IBPA-range (Figure 6).

3.2 Phase-Mistuning

In the tuned case all blades oscillate with identical frequency ω and the same phase difference σ between each pair of adjacent blades as defined by Equation (8). A simple way of mistuning the system is to slightly change the phase of one of the blades, so that the IBPA is not constant over the blade row. For example, Blade 1 in Figure 2 can be mistuned by a phase shift $\Delta\sigma$, so that

$$\begin{bmatrix} h_x^{(1)}(t) \\ h_y^{(1)}(t) \\ \alpha^{(1)}(t) \end{bmatrix} = a \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_\alpha / c \end{bmatrix} e^{i(\omega t + \sigma + \Delta\sigma)} \quad (10)$$

The phase difference between blade 0 and blade 1 then becomes $\sigma + \Delta\sigma$, and $\sigma - \Delta\sigma$ between blade 1 and blade 2, whereas all other IBPAs remain at the original value σ. Although this type of phase-mistuning is somewhat artificial since a real system would pick up its own phase differences based on the flow and structural conditions, it serves the purpose of studying the potential effect of mistuned phase on the motion of flutter.

Figures 7 and 8 show the calculated damping coefficient Ξ as a function of σ for this type of phase-mistuning and $\Delta\sigma = +10°$. The phase difference σ in this plot is the *average* IBPA of the cascade, which is the same as in the tuned case. Using only four passages limits the choice for σ to $0°, 90°, 180°$ and $270°$. For these inter-blade phase angles the results for the tuned case, already shown in Figure 6, are also plotted in Figures 7 and 8.

It is obvious that, due to mistuning, the damping coefficient Ξ varies for different blades. However, only blade 0 and blade 1 show significant changes with respect to the tuned case. Mistuning blade 1 impacts principally itself and its immediate neighbors, i.e. blade 0 and blade 2. The blade order is such that blade number i+1 is adjacent to the suction side of blade number i (see Figure 2). The flow on the suction surfaces of the blades is more sensitive to changes of conditions than that on the pressure surfaces. Consequently, it is blade 0 and blade 1 that will be affected most by the mistuned motion of blade 1. The flow around blade 2 is not affected very much since it has only its less sensitive pressure side facing blade 1.

The changes in damping coefficient and therefore stability go in opposite directions for blade 0 and blade 1, as shown by Figures 7 and 8. Blade 0 mainly feels a phase difference of $\sigma + 10°$ at its suction side. At any σ the mistuned value of Ξ qualitatively changes towards the tuned value for $\sigma + 10°$. The damping coefficient of blade 1, being dominated by a phase difference $\sigma - 10°$ on the suction side, shifts to the tuned value at $\sigma - 10°$. This qualitative behavior also holds for a negative $\Delta\sigma$ as demonstrated by the computational results shown in Figures 9 and 10. It can be seen that $\Delta\Xi$ is small at relative extrema and large in between. In general, both the absolute value and sign of $\Delta\Xi$ appear to depend on the imposed mistuning phase difference $\Delta\sigma$ and the slope of the tuned stability curve. If $\Delta\sigma$ is considerably small, the effect of phase-mistuning can be expressed by the following:

$$\Delta\Xi \sim \left.\frac{\partial\Xi}{\partial\sigma}\right|_{tuned} \cdot \Delta\sigma$$

3.3 Frequency-Mistuning

Another way to introduce mistuning is to let the blades oscillate with slightly different frequencies. In a first study this is done again with a single blade out of four, i.e. blade 1 in Figure 2 vibrates with a slightly higher frequency than the other blades.

The motion of the mistuned blade 1 is expressed as

$$\begin{bmatrix} h_x^{(1)}(t) \\ h_y^{(1)}(t) \\ \alpha^{(1)}(t) \end{bmatrix} = a \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_\alpha / c \end{bmatrix} e^{i(\omega t + \Delta\omega t + \sigma)} \quad (11)$$

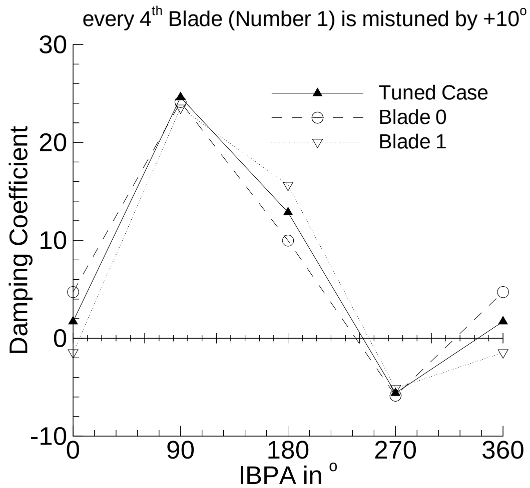

Figure 7: Damping coefficient vs. IBPA for blade 0 and 1. Blade 1 is mistuned in phase by +10°.

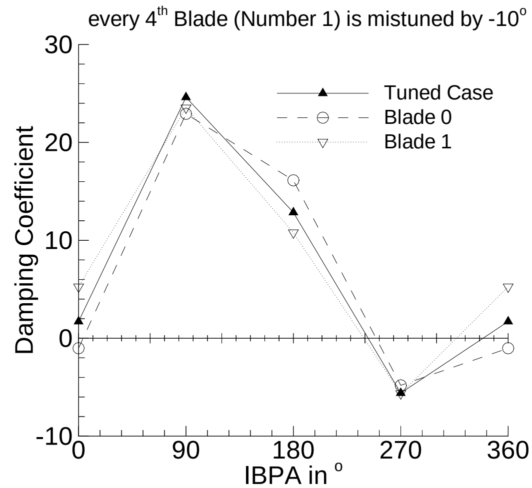

Figure 9: Damping coefficient vs. IBPA for blade 0 and 1. Blade 1 is mistuned in phase by −10°.

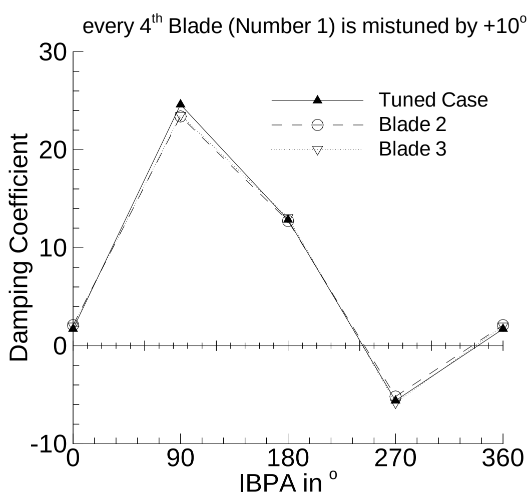

Figure 8: Damping coefficient vs. IBPA for blade 2 and 3. Blade 1 is mistuned in phase by +10°.

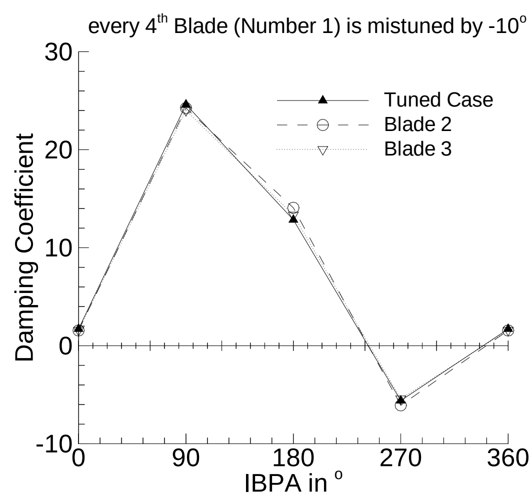

Figure 10: Damping coefficient vs. IBPA for blade 2 and 3. Blade 1 is mistuned in phase by −10°.

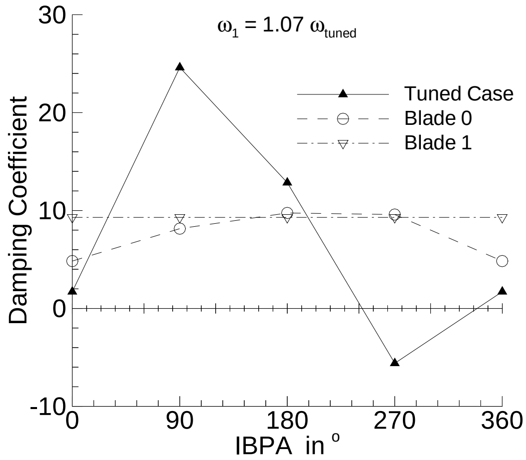

Figure 11: Damping coefficient vs. IBPA for blade 0 and 1. Blade 1 is mistuned in frequency by +7%.

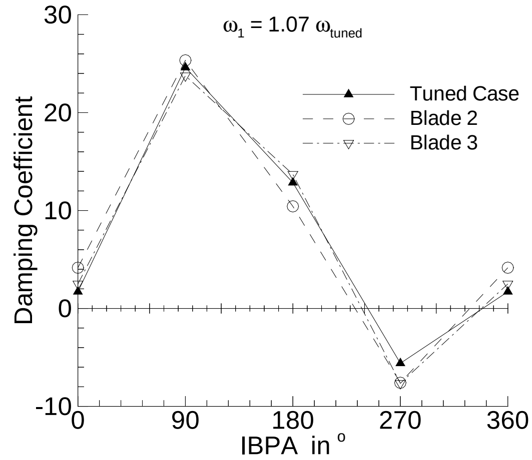

Figure 12: Damping coefficient vs. IBPA for blade 2 and 3. Blade 1 is mistuned in frequency by +7%.

For $\Delta\omega = \frac{1}{14}\omega$ the influence on the damping coefficient is shown in Figures 11 and 12. Again only blade 0 and blade 1 are significantly affected. The damping of the mistuned blade 1 is independent from the inter-blade phase angle. This is not surprising, because the concept of a constant IBPA does not apply any more to blade 1. The phase differences of blade 1 with respect to the three other blades are continuously changing in time. After one period of the mistuned system, blade 1 has gone through all IBPAs from 0° to 360°. Its damping coefficient is close to the tuned Ξ averaged over the IBPA, which is a positive (stable) value. Blade 0 still has constant IBPAs with respect to blades 2 and 3. The permanently changing phase difference with respect to its suction side neighbor (blade 1), however, weakens the IBPA dependence of blade 0. It becomes stable at any IBPA. Blades 2 and 3 hardly change compared to the tuned case.

The positive effect on the mistuned blade itself and on one of its neighbors leads to the idea of alternate mistuning: every second blade in the row is mistuned in the same way, i.e.

$$\begin{bmatrix} h_x^{(m)}(t) \\ h_y^{(m)}(t) \\ \alpha^{(m)}(t) \end{bmatrix} = a \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_\alpha / c \end{bmatrix} e^{i(\omega t + \Delta\omega t + m\sigma)}, m = 1, 3, 5, ... \tag{12}$$

Figure 13 shows the result for the same amount of frequency mistuning as in the previous case, but here imposed on both blade 1 and blade 3. In this case, all blades are stable with little dependence on the IBPA. The reason for this behavior can be explained as follows. Because the frequency of every other blade in the row is perturbed from its nominal frequency, each blade is vibrating at a frequency different from its two immediate neighbors. There is not a fixed IBPA between any blade and its immediate neighbors. If we neglect the influence by far neighbors, each blade can be viewed as the mistuned blade in the single blade mistuning situation discussed earlier and shown in Figure 11, with either a positive or negative frequency perturbation. Consequently, the damping coefficient for this blade would be independent of the nominal IBPA and would take the phase average of the tuned values. For this case it becomes positive and therefore the blades becomes stable. Since this is true for every blade in the row, we expect that all blades become stable with this

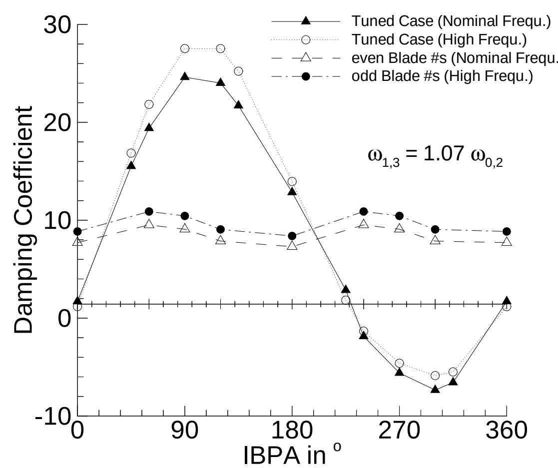

Figure 13: Damping coefficient vs. IBPA. Odd numbered blades are mistuned in frequency by +7%.

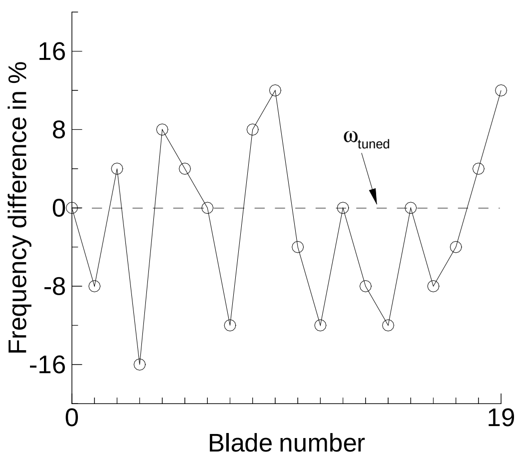

Figure 14: Randomlike frequency distribution over 20 blades.

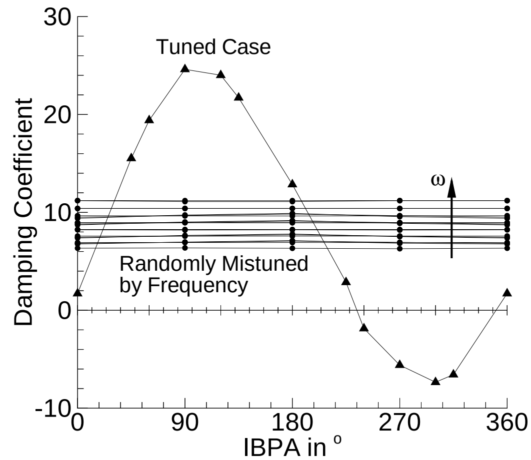

Figure 15: Damping coeffcient for configuration shown in Figure 14.

alternate mistuning pattern. Indeed, this is confirmed by the computational result shown in Figure 13. The two sets of damping coefficients corresponding to the odd and even numbered blades are both positive and almost independent of the nominal IBPA. The slight differences between the two sets are due to the frequency difference between the blades. The two tuned cases shown in Figure 13 indicate that in the tuned case the blade motion becomes more stable with increasing frequency at almost all inter-blade phase angles. The odd numbered blades can be viewed as mistuned from its neighbors with a positive frequency perturbation while the even numbered blades can be viewed as mistuned with a negative frequency perturbation. As such, the phase averaged damping coefficient for the odd numbered blades becomes higher than that for the even numbered blades.

The above findings agreed qualitatively with the experiment data by Nowinski and Panovsky [6] even though our computations and analysis were performed independently, for a different cascade and without the knowledge of the experimental work.

Although the effect of far neighbors is small, it is still noticeable in Figure 13. The damping coefficients are not exactly constant. Instead, they exhibit small variations with the nominal IBPA at twice the frequency as that in the tuned case. This is because there is still a fixed phase difference between each blade and its second neighbors, and that phase difference is exactly twice the nominal IBPA.

The effect of alternate mistuning in this case can be thought of as splitting the blade row into two staggered tuned systems damping each other. The effect of frequency-mistuning should not be thought of as an introduction of new damping. It is rather taking away the IBPA specific influence of neighboring blades. Whether this leads to more or less stability than in the tuned case depends on the phase averaged damping coefficient at the given frequency. It may be expected, that this type of mistuning might introduce absolute instability if the tuned blades exhibit instability over a majority of the IBPA range. We have yet to find such a test case to verify this conjecture.

Aerodynamic decoupling of adjacent blades, and in this case stabilization, is achieved by alternating the frequency. However, there is no need for the frequency to follow a certain pattern throughout the cascade, as long as immediate neighbors oscillate with different frequencies. The effect is expected to be even stronger if we apply a randomlike frequency distribution on a large number of blades (Figure 14). The damping coefficient for this configuration is shown in Figure 15. The horizontal lines are the calculated damping coefficients of the individual blades. There is almost no dependence on the IBPA, because blades oscillating at the same frequency are too distant to influence each other. It is also evident from Figure 15 that the higher the frequency, the larger is the damping coefficient, as in the case in Figure 13.

It is clear from the above discussion that frequency mistuning may stabilize a blade row. The question arises as to how much mistuning, i.e. $\Delta\omega$, is needed. Even if $\Delta\omega$ is infinitesimally small, the phase difference between adjacent blades would still be constantly changing with time although at a slower rate. The damping coefficient is defined as

$$\Xi = -\frac{C_W}{\pi h^2}, \qquad C_W = \frac{T_{tuned}}{T} \int_0^T C_h dh \qquad (13)$$

where h is the translational displacement, C_W is the work coefficient, C_h the force coefficient in h direction, T_{tuned} the period of the tuned case, and T the overall period of the system. Our computations show that the beneficial effect of frequency mistuning is not affected by the frequency difference $\Delta\omega$ if one looks only at the damping coefficient defined by Equation (13). This obviously cannot be true when $\Delta\omega$ goes to zero in the limit of the unstable tuned case. To understand this situation we must recognize that the damping coefficient as calculated from Equation (13) in an uncoupled energy method is only a measure of stability over the overall period T. In the case of frequency mistuning the overall period T is different from T_{tuned} because the blades are oscillating with different frequencies. The overall period is determined by the smallest frequency difference that appears in the system. With decreasingly small frequency differences $\Delta\omega$, the overall period becomes very long and the phase differences between adjacent blades change very slowly. It is conceivable that an originally unstable blade may absorb a large amount of energy from the flow during an initial period of time when the phase angle differences between blades have not yet changed much from the initial nominal value, although the total energy absorption will eventually become negative over the long overall period T. In that case the damping coefficient calculated by Equation (13) would not be meaningful, since the blade may already have broken before time T is reached.

To verify the above point we perform calculations of the alternate mistuning case with $\Delta\omega$ = 1.0%, 0.5%, 0.25% and 0% (tuned case) at the most critical IBPA of 270°. Figure 16 shows the instantaneous work coefficient done on blade 0 versus time for the different mistuning cases. For clarity, only the envelope, i.e. the temporal maximum of work, is plotted, except for the case when $\Delta\omega$ = 1.0%. All cases start off with amplification. The total maximum is reached at about the first quarter period, and its magnitude continuously increases with decreasing frequency difference. This maximum may exceed the allowable deformation work and the blade will break although the theoretical damping coefficient by Equation (13) is still a positive number. Short of using a coupled fluid-structure interaction approach such as that described by He [11], it is not possible to obtain a quantitative measure of the needed $\Delta\omega$, except the above qualitative guidance.

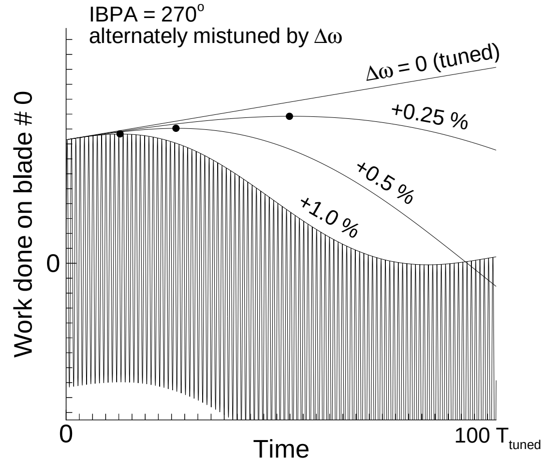

Figure 16: Work done on blade 0 vs. time, for various frequency differences.

4 Conclusions

A computational method for predicting flutter of turbomachinery cascades with mistuned blades is presented. The method is based on solving the unsteady Euler/Navier-Stokes equations through multiple blade passages on a parallel computer. Each individual blade is capable of moving with its own independent frequency and also phase angle, therefore allowing flutter predictions with either frequency or phase-mistuning. Computations for a turbine blade row show that mistuning in phase has relatively small effect on the flutter characteristics of the blade row. On the other hand, frequency-mistuning can have significant influence on the damping coefficient of the mistuned blade and its adjacent neighbor. The result is a damping coefficient averaged over the complete IBPA range (0° - 360°) because the actual phase differences between the mistuned blade and its adjacent blades are constantly changing within that range due to the frequency difference. If the blade is stable over most of the IBPA range in the tuned case, the blade will then become stable in an overall sense in the mistuned case. When this effect is made use of in constructing a blade row with alternately or randomly mistuned blades, it is found that frequency-mistuning may stabilize all blades over the whole effective IBPA range. Random mistuning eliminates entirely the dependence of the aerodynamic damping on the inter-blade phase angle. The minimum amount of mistuning needed for stability is also investigated. It is identified that a blade may absorb too much energy from the flow if there is not enough frequency difference, so that it may fail in a short time even though the overall aerodynamic damping is positive over a long period. The studies in this paper, however, are limited to the use of the energy method, which is only valid for blades with large mass ratios. More definite studies that include the effect of frequency- and phase-shift of the structural system and accurate prediction of the blade vibration amplitude must be performed with a coupled fluid-structure interaction method.

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